Mahidol University International College
Final Examination
ICMA/ICNS 102, ICMA 106: Principle of Mathematics, Calculus I
First Trimester 2014-2015

Problem 1. (10 points)

Let $f(x) = \dfrac{1}{\sqrt{1-x}}$.

(a) Find the local linear approximation of $f(x)$ at $x = 0$.

(b) Using the formula obtained in part (a), or otherwise, approximate

$$\frac{1}{\sqrt{0.9}}, \frac{1}{\sqrt{0.98}}, \frac{1}{\sqrt{1.2}}.$$

Your answers must be in decimals.

Problem 2. (10 points)

(a) Find the differential dy if $y^2 = \sec(2x)$.

(b) A 10-foot ladder is leaning against the wall and on a slippery surface. If the bottom of the ladder is sliding away from the wall at a rate of 4 ft/sec. At the moment where the bottom of the ladder is 8 feet from the wall, how fast is the top of the ladder moving down the wall?

Problem 3. (10 points)Let $f(x) = (x-1)^2(x-2)$.

(a) Find the intervals on which f is increasing and the intervals on which f is decreasing. Find all relative extrema.

(b) Find the intervals on which f is concanve up and the intervals on which f is concave down.

(c) Find the coordinates of all inflection points.

Problem 4. (10 points)

(a) Find $\dfrac{dy}{dx}$ if $x^3 + y^3 = 3xy$.

(b) Explain what it means by "$x = c$ is a stationary point of a function $f(x)$".

(c) Suppose that f is a continuous function such that $f'(x) = x^4 - x^3 + 2x^2 - 2x$.

i. Show that $x = 0$ and $x = 1$ are stationary points of f.

ii. Determine whether f has a relative maximum or relative minimum at $x = 0$ and $x = 1$.

Problem 5. (10 points)

Let $f(x) = x(x-8)^3$. Sketch a graph of this polynomial and label the coordinates of the intercepts, relative extrema, and inflection points.

Problem 6. (10 points)

Find the absolute maximum and minimum values of $f(x) = \dfrac{x}{x^2+4}$ on $[0, 3]$.

Problem 7. (10 points)

A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $y = 12 - x^2$. For all such rectangles, what are the dimensions of the one with largest area. Justify your answer.

Problem 8. (10 points)

(a) Complete the statement of the Mean-Value Theorem.

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

Then ....

(b) Verify that the hypotheses of the Mean-Value Theorem are satisfied on the given interval, and find all values in that interval that satisfy the conclusion of the theorem.

$$f(x) = 3x^3 - 12x; [0, 1].$$

Problem 9. (10 points)

(a) Evaluate the integral:

$$\int \frac{1 - x^2}{\sqrt{x}}\, dx.$$

(b) Evaluate the integral:

$$\int (\sin x - \cos x)^2\, dx.$$

(c) Solve the initial-value problem:

$$\frac{dy}{dx} = \frac{1}{\cos^2 x}, \quad y\left(\frac{\pi}{4}\right) = 3.$$

Problem 10. (10 points)Evaluate the integrals using appropriate substitutions.

a) $\int t^2 \sqrt{t^3 + 12}\, dt$

b) $\int \frac{\csc^2(\sqrt{x})}{\sqrt{x}}\, dx$

Problem 11. (10 points)Consider $f(x) = x^2 + 2$ on $[0, 1]$.

a) Divide the specified interval into $n = 4$ subintervals of equal length, Δx, and then compute

$$\sum_{k=1}^{4} f(x_k^*)\Delta x$$

with x_k^* as the left endpoint of each subinterval. Illustrate this with a graph of f that includes the rectangles whose areas are represented in the sum.

b) Evaluate the area under the curve $f(x) = x^2+2$ and the interval $[0, 1]$ by using the **net signed area**

$$A = \lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x.$$

Problem 12. (10 points)Sketch the region whose signed area is represented by the definite integral, and evaluate the integral using an appropriate formula from geometry, where needed.

a) $\int_0^3 |2 - x|\, dx$

b) $\int_{-\pi/4}^{\pi/4} \cos x\, dx$

Problem 13. (10 points)Evaluate the integrals

a) $\displaystyle\int_1^{16} \frac{1}{x\sqrt[4]{x}}\,dx$

b) $\displaystyle\int_{\pi/6}^{\pi/3} \left(x^2 + \frac{2}{\sin^2 x}\right)\,dx$

Problem 14. (10 points)

a) State the Mean-Value Theorem for Integration.

b) Let $f(x) = x^2$. Find all values of x^* in $[0, 2]$ such that

$$\int_0^2 f(x)\,dx = f(x^*)(2-0).$$

Problem 15. (10 points)Let $F(x) = \displaystyle\int_{\pi/2}^{x} \sqrt{\sin(t)+9}\,dt$. Find

a) $F(\pi/2)$

b) $F'(\pi)$

Problem 16. (10 points)Evaluate the integrals

a) $\displaystyle\int_0^1 \frac{x^2+2}{(x^3+6x+1)^2}\,dx$

b) $\displaystyle\int_0^{\sqrt{\pi}} 10x\sin(x^2)\,dx$

Problem 17. (10 points)

Sketch the region enclosed by the following curves and find its area.

$$y = x^2, y = \sqrt{x}.$$