



Mahidol University International College

Final Exam, Trimester 2, 2014-2015 ICMA/ICNS 102, ICMA 106 Principles of Mathematics, Calculus I

Saturday, 28 March 2015 10:00 to 11:50

70 points, 35%

Directions: Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section number in the space provided on every page. The last page can be used for scratch-work.

Question 1 (Total: 7 points)

out of 7 points

1.1 Let f be a differentiable function. Write down the formula for the local linear approximation of $f(x)$ near $x = a$. (2 points)

1.2 Use local linear approximation to approximate the value of $\frac{1}{\sqrt{(1.9)^2 + 5}}$. Write your final answer in the form $\frac{n}{135}$ where n is an integer to be found. (5 points)

Question 2 (Total: 11 points)

out of 11 points

Let $f(x) = x^4 - 4x^3$

- 2.1 Find the intervals on which f is increasing and the intervals on which f is decreasing.
Find all relative extrema. (4 points)

- 2.2 Find the intervals on which f is concave up and the intervals on which f is concave down.
Find coordinates of all inflection points. (4 points)

2.3 Sketch the graph of $y = f(x)$. Label the coordinates of all relative extrema, inflection points, x -intercepts and y -intercepts. (3 points)

Question 3 (Total: 3 points)

out of 3 points

Let $f(x) = 8x^6 + 45x^4 - 24x^2$. Show that $x = 0$ is a stationary point of f . Determine whether this is a relative maximum, relative minimum, or neither. (3 points)

Question 4 (Total: 6 points)

out of 6 points

Find the absolute maximum and minimum values of $f(x) = \sqrt[3]{x^2 + 4}$ on $[-2, 1]$. (6 points)

Question 5 (Total: 3 points)

out of 3 points

What function $f(x)$ satisfies

(3 points)

$$\int (x + 1)f(x) dx = \frac{1}{9}x^9 + \frac{1}{8}x^8 + C?$$

Question 6 (Total: 11 points)

out of 11 points

Evaluate the integrals:

6.1 $\int \sqrt{x}(1 + x^2) dx$ (3 points)

6.2 $\int \frac{\sec x - \cos^2 x}{4 \cos x} dx$ (4 points)

6.3 $\int_0^3 \frac{x}{\sqrt{x+1}} dx$ (Write your final answer in fraction form.) (4 points)

Question 7 (Total: 8 points)

out of 8 points

Solve the initial-value problems:

7.1 $\frac{dy}{dx} = x\sqrt[3]{x^2}, \quad y(1) = 2.$

(4 points)

7.2 $\frac{dy}{dx} = (\sin x + \cos x)^2, \quad y\left(\frac{\pi}{4}\right) = \frac{\pi}{2}.$

(4 points)

Question 8 (Total: 6 points)

out of 6 points

Let $f(x) = 1 + 2x$.

Divide the interval $[1, 5]$ into $n = 4$ subintervals of equal length and then compute

$$\sum_{k=1}^4 f(x_k^*) \Delta x$$

with x_k^* as the left endpoint of each subinterval. Illustrate with the graph of f that includes the rectangles whose areas are represented in the sum. (6 points)

Question 9 (Total: 10 points)

out of 10 points

Given function

$$g(x) = \int_2^x \sqrt{1+t^3} dt,$$

9.1 What is $g'(x)$? (3 points)

9.2 What is an equation of the tangent line to the curve $y = g(x)$ at $x = 2$? (4 points)

9.3 What is $g''(x)$? (3 points)

Question 10 (Total: 5 points)

out of 5 points

Find the area of the shaded region. Write your final answer in fraction form. (5 points)

