Mahidol University International College

ICMA102 | ICNS102 | ICMA106
Final Exam, Trimester 3, 2014-15

Saturday 18 July 2015	8:00 – 9:50	73 points, 35%

Name: ______________________________ I.D.: ____________________

Section: ____________ Seat: ____________

1. (a) Find the local linear approximation of $f(x) = \sin x$ near $x = \dfrac{\pi}{3}$. **[3]**

   (b) Now approximate the value of $\sin\left(\dfrac{7\pi}{18}\right)$. (Note that $\dfrac{7\pi}{18} \approx \dfrac{6\pi}{18} = \dfrac{\pi}{3}$.) **[2]**

2. For what values of the constants a and b is the point $(1, 3)$ a critical point of the curve $y = ax^3 + bx^2$.? **[4]**

3. A curve is such that

$$\frac{dy}{dx} = 2(3x+4)^{3/2} - 6x - 8.$$

(a) Verify that the curve has a stationary point when $x = -1$. **[1]**

(b) Use an appropriate test to determine the nature of the stationary point at $x = -1$. **[3]**

(c) It is now given that the stationary point on the curve has coordinates $(-1, 5)$. Find the equation of the curve. **[4]**

4. Let $f(x) = x^3 - x^2 - 5x + 7$.

   (a) Determine the intervals on which f is increasing and intervals on which f is decreasing. **[3]**

   (b) Find the coordinates of the relative *maximum* point on the curve $y = f(x)$. **[2]**

   (c) Find the interval on which f is concave *down*. **[2]**

   (d) Find the coordinates of the inflection point on the curve $y = f(x)$. **[1]**

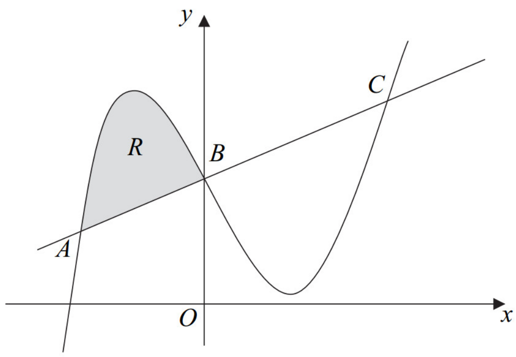
5. The diagram on the right shows a curve and a line which intersect at the points A, B and C. The curve has equation

$$y = x^3 - x^2 - 5x + 7$$

and the straight line has equation

$$y = x + 7.$$

The point B has coordinates $(0, 7)$.



(a) Find the coordinates of the point A. **[3]**

(b) Find $\int (x^3 - x^2 - 5x + 7)\, dx$ **[2]**

(c) Find the area of the shaded region R. **[3]**

6. Let $f(x) = \dfrac{x+1}{x^2+3}$. Find all absolute extrema of $f(x)$ on $[0, 3]$. **[5]**

7. Let $F(x) = \displaystyle\int_0^x \sqrt[3]{t+1}\, dt$.

   (a) Find $F(0)$ and $F'(7)$. **[2]**

   (b) Find $F'''(0)$. **[2]**

8. (a) Complete the following statement of the Mean Value Theorem for Integrals: **[1]**

Let f be a continuous function on $[a, b]$. Then

. . . .

(b) Let $f(x) = \dfrac{4}{x^2}$ be defined on the interval $[-4, -2]$. Explain why $f((x)$ is continuous on $[-4, -2]$. Then find the value of c that satisfies the conclusion for the Mean Value Theorem for Integrals. **[4]**

9. Find $\int x^{\frac{1}{3}} \sqrt{x^{\frac{2}{3}} + 1}\, dx$. **[5]**

10. Sketch the region enclosed by the curves $y = (x-2)^2$, $y = x - 2$, and $x = 0$ and find its area. **[5]**

Do not write on this margin

11. Find $\int_0^{\pi/3} \sin(t)\cos^2(t)\,dt$. **[4]**

12. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible. **[5]**

13. (a) Estimate the area under the graph of $f(x) = (1+x)^2$ from $x = -1$ to $x = 2$ using *three* rectangles and *right* endpoints. Then improve your estimate by using *four* rectangles. **[5]**

(b) What is the exact area under the graph of $f(x) = (1+x)^2$ from $x = -1$ to $x = 2$? **[2]**