Mahidol University International College

Principles of Mathematics / Calculus I
ICMA102 / ICNS102 / ICMA106
Final Exam, Trimester 1, 2015-16

Saturday 12 December 2015 | 12:00 – 13:50 | 80 points, 35%

Name: ______________________ I.D.: ______________

Section: __________ Seat: __________

Directions: This exam paper contains 12 pages and 12 questions. Show all your work clearly. A calculator is NOT allowed for this exam. Scratch-paper can be found in the last sheet, which can be removed.

1. (a) Let f be a differentiable function. Write down the formula for the local linear approximation of $f(x)$ near $x = a$. **[2]**

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(b) Use local linear approximation to approximate the value of $\sqrt[3]{(1.9)^2 + 4}$. Write your final answer in the form $\dfrac{n}{30}$ where n is an integer to be found. **[4]**

2. The following questions refer to the function $f(x) = \frac{1}{3}x^3 - 3x$.

   (a) Find all intercepts of the graph of this function. **[2]**

   (b) Determine intervals where the function is increasing and where the function is decreasing. Determine the points (x, y) of all relative extrema. Specify which point is a relative maximum and which is a relative minimum. **[3]**

   (c) Determine intervals where the graph of function is concave up and where the graph is concave down. Find all inflection points. **[3]**

(d) Sketch the graph of this function. Label important points (intercepts, relative extrema, and inflection points) on the graph. **[3]**

3. Let $f(x) = 3x(x+4)^{\frac{2}{3}}$.

(a) Find $f'(x)$. Write your answer in the form $\dfrac{ax+b}{(x+4)^{1/3}}$. **[3]**

(b) Find all absolute extrema of $f(x)$ on the interval $[-5, -3]$. **[3]**

4. Verify that the hypotheses of the *Mean Value Theorem for Derivatives* are satisfied for the function $f(x) = \dfrac{x}{x-1}$ on the interval $[4, 5]$. Find all value(s) of c in that interval that satisfy the conclusion of the theorem. Note that $\sqrt{3} \approx 1.732$. **[4]**

5. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing? **[5]**

6. A 20 inch wire is cut in two and shaped into two squares. What is the minimum possible sum of the two areas? Justify why you get the minimum. **[5]**

7. Evaluate the following integrals.

(a) $\displaystyle\int_0^{-\pi/2} (2x - \cos x)\, dx$ **[3]**

(b) $\displaystyle\int \frac{2x + \frac{3}{x}}{3\sqrt{x}}\, dx$ **[3]**

(c) $\displaystyle\int 2\sec x(3\sec x + 4\tan x)\, dx$ **[3]**

(d) $\displaystyle\int x^5\sqrt{x^6 + 9}\, dx$ **[3]**

8. Let $f(x) = \int_1^x (t+1)\sqrt{t-1}\,dt$. Find

(a) $f(1)$ **[2]**

(b) $f'(5)$ **[2]**

(c) $f''(5)$ **[2]**

(d) $f(2)$ **[3]**

9. Let $f(x) = \begin{cases} -x-2 & x < -2 \\ \sqrt{4-x^2} & -2 \leq x < 0 \\ \sin(x)+2 & x \geq 0. \end{cases}$

(a) Determine whether f is continuous at $x = -2$. **[3]**

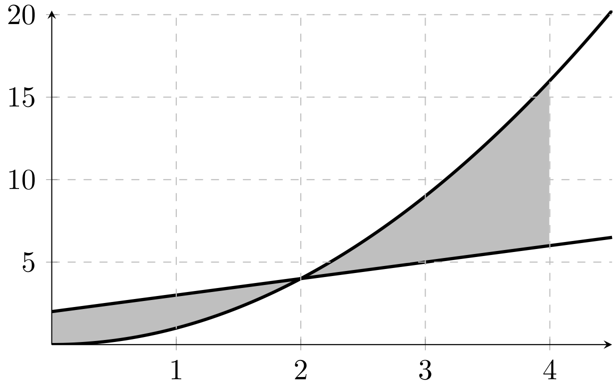
(b) Find $\lim_{x\to\pi} \dfrac{f(x)}{x}$. **[2]**

(c) Evaluate $\int_{-2}^{\pi} f(x)\,dx$. Write your final answer in the form $a + b\pi$ where a, b are integers to be determined. **[4]**

10. Explain why the *Mean Value Theorem for Integrals* applies to the function $f(x) = x^2 + x$ on the interval $[-12, 0]$. Find all value(s) of c that satisfy the conclusion of the theorem. **[4]**

11. Let $f(x) = \sin x$. Estimate the area under the graph of $f(x)$ over the interval $[0, \pi]$ using *four* rectangles and *right* endpoints. Write your final answer in the simplest form. Sketch the graph of $f(x)$ and draw the rectangles to indicate the areas you are calculating. **[4]**

12. Below are the graphs of $f(x) = x^2$ and $g(x) = x + 2$ on the same plane. Determine the area of the shaded region. **[5]**



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