Mahidol University International College
Midterm Examination
ICMA/ICNS 102, ICMA 106: Principle of Mathematics, Calculus I
Second Trimester 2015-2016
(70 points = 35%)

Instructions. The exam consists of 9 main problems with points indicated in each problem. Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section instructor's name in the space provided on every page. Two pages of scratch-paper are provided in the back of this exam.

Name.....I.D. .....Instructor's Name.....

SCORE

Problem 1.

a) $\lim_{x\to+\infty}\left(\sqrt{x-2}-\sqrt{2x}\right)$ (2 points)

b) Find the value of $c \in [-5, 3]$ so that the function $f(x) = 2x$ satisfies the mean value theorem for integrals. (2 points)

Name.....I.D. .....Section.....

SCORE

Problem 2.

Find the absolute maximum and minimum values of $f(x) = 5 + 3x^2 - 2x^3$ on $[0, 3]$.

(5 points)

Name.....I.D. .....Section.....

SCORE

Problem 3.

Let $f(x) = \dfrac{x-1}{x^2}$. Then $f'(x) = \dfrac{2-x}{x^3}$ and $f''(x) = \dfrac{2x-6}{x^4}$.

(a) Find the x and $y-$intercepts as points (if they exist). (1 point)

(b) Find the intervals on which $f(x)$ is increasing and on which it is decreasing. Find all relative extrema as points. (3 points)

(c) Find the intervals on which $f(x)$ is concave up and on which it is concave down. Find all inflection points. (3 points)

(d) Find all asymptotes of $f(x)$. (2 points)

(e) Sketch the graph of $f(x)$, labelling all the points and asymptotes you found. (4 points)

Name.....I.D. .....Section.....

SCORE

Problem 4.

A right triangle has hypotenuse 5 cm. Find the perimeter of the triangle that is a maximum. Justify you answer using the 1^{st} or 2^{nd} derivative test.

(5 points)

Name.....I.D. .....Section.....

SCORE

Problem 5.

Find the antiderivative F of f satisfying the given condition.

a) $f(x) = 5x^3 - 2x^4, F(0) = 1$ (3 points)

b) $f(x) = 1 + 3\cos(x),\ F\left(\frac{\pi}{2}\right) = \pi + 3$ (3 points)

Name.....I.D. .....Section.....

SCORE

Problem 6.

Let $f(x) = 3x + 2$. Divide the interval $[0, 1]$ into n subintervals of equal length and then compute

$$\sum_{k=1}^{n} f(x_k^*)\Delta x$$

with x_k^* as the right endpoint of each subinterval. Find the value of the sum above as n approaches infinity. Explain what the limit represents in symbols and geometrically.

Note You may want to consult the following formulas:

- $\sum_{k=1}^{n} 1 = n$
- $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
- $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$.

(5 points)

Name.....I.D. .....Section.....

SCORE

Problem 7.

Evaluate the integrals (a - f):

a) $\displaystyle\int \frac{3-x^3}{\sqrt{x}}\,dx$ (3 points)

b) $\displaystyle\int \sec t(\sec t+\tan t)\,dt$ (3 points)

c) $\displaystyle\int_0^1 x(1-x)^2\,dx$ (3 points)

d) $\displaystyle\int \frac{3x}{\sqrt{x^2+3}}\,dx$ (3 points)

e) $\displaystyle\int \csc x \cot x \sqrt{\csc x}\,dx$ (3 points)

f) $\displaystyle\int_0^{\pi/2} \frac{1+\cos(2t)}{2}\,dt$ (3 points)

Name.....I.D. .....Section.....

SCORE

Problem 8.

Given function

$$g(x) = \int_{\frac{\pi}{2}}^{x} \sqrt{3 + \sin^2 t}\, dt,$$

a) What is $g'(x)$? (3 points)

b) What is an equation of the tangent line to the curve $y = g(x)$ at $x = \dfrac{\pi}{2}$? (3 points)

c) What is $g''(x)$? (3 points)

Name.....I.D. .....Section.....

SCORE

Problem 9.

Find the area of the region enclosed by the curves $y = x^2$ and $y = \sqrt{x}$. (5 points)

(Scratch-paper)

(Scratch-paper)