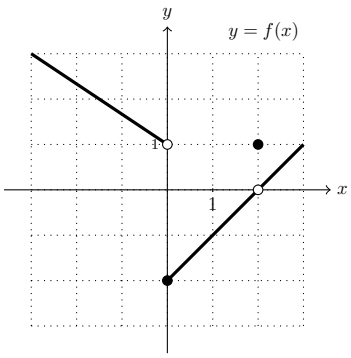
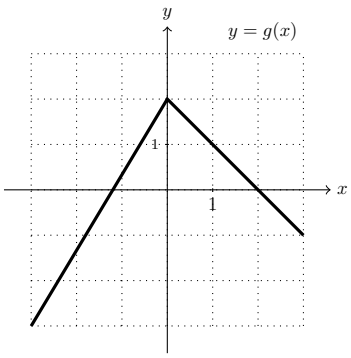
ICMA/ICNS 102, ICMA 106: Principle of Mathematics, Calculus I
Midterm Examination First Trimester 2014-2015

Problem 1. (4 points)

(a)-(d) Use the graphs of f and g below to find the limits that exist. If the limit does not exist, explain why.





(a) $\lim_{x\to 2}\big[f(x)+g(x)\big]$ (1 point)

(b) $\lim_{x\to 2}\dfrac{1+g(x)}{f(x)}$ (1 point)

(c) $\lim_{x\to 0^-}\big[f(x)+g(x)\big]$ (1 point)

(d) $\lim_{x\to 2^+}\sqrt{f(x)}$ (1 point)

Problem 2. (5 points)

(a)-(d) Find the limits.

(a) $\lim_{x\to 2}\dfrac{x^2-4x+4}{x^2+x-6}$ (1 point)

(b) $\lim_{x\to 9}\dfrac{x-9}{\sqrt{x}-3}$ (1 point)

(c) $\lim_{x\to 2}\dfrac{2x}{x^2-4}$ (1 point)

(d) $\lim_{x\to 1}\dfrac{x^2}{|x-1|}$ (2 points)

Problem 3. (5 points)

(a)-(b) Find the limits.

(a) $\lim_{x\to\infty}\dfrac{3x^2-4x+4}{x-6-x^2}$ (1 point)

(b) $\lim_{x\to\infty}(\sqrt{x^2+3x}-x)$ (2 points)

(c) Use the Intermediate Value Theorem to show that the equation

$$x^{10}+4x-x^9-2=0$$

has a root in the interval $[0,1]$. (2 points)

Problem 4. (3 points)

Find all the values of x at which $f(x)$ is not continuous. Justify your answer.

$$f(x) = \begin{cases} 7 + \dfrac{16}{x}, & x < 4, \\ 2x + 2, & 4 \le x < 6 \\ 10 + \dfrac{4}{x-5} & x \ge 6. \end{cases}$$

(3 points)

Problem 5. (4 points)

(a) Find the limits.

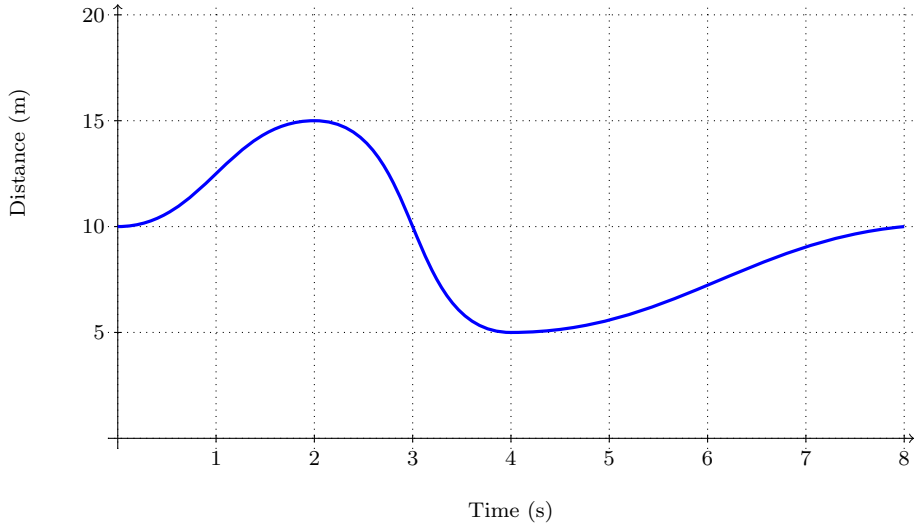
i. $\lim_{x \to 0} \cos(\pi - \sin x)$ (1 point)

ii. $\lim_{x \to +\infty} (3\cos^2 x + 3\sin^2 x)$ (1 point)

(b) If $\lim_{x \to 0} \dfrac{5x + \sin(kx)}{x} = 7$, find the value of k. (2 points)

Problem 6. (3 points)

(a) The accompanying figure shows the position versus time curve for a certain particle moving along a straight line.



i. Find the average velocity over the interval $2 \le t \le 4$. (1 point)

ii. Circle (all that apply) the value of t at which the instantaneous velocity is zero. (1 point)

A) $t = 1$ B) $t = 2$ C) $t = 3$ D) $t = 4$ E) $t = 5$

iii. Which of the following best approximates the instantaneous velocity when $t = 1$ s. (1 point)

A) -1 m/s B) 0 m/s C) 4 m/s D) 100 m/s E) -8 m/s

Problem 7. (5 points)

(a) Complete the following statement: The derivative of f with respect to x is the function $f'(x)$ defined by the formula (1 point)

$f'(x) =$ .....

(b) Let $f(x) = x^2 + 2x$. Use the definition of the derivative in part (a) to find $f'(x)$. (2 points)

(c) Given that the tangent line to $y = f(x)$ at the point $(2, 4)$ passes through the point $(-1, 5)$, find $f'(2)$. (2 points)

Problem 8. (5 points)

(a) Find $\dfrac{dy}{dx}$.

i. $y = -5x^{-6} + 3\sqrt{x}$ (1 point)

ii. $y = \dfrac{\sqrt[3]{8x} + 5x}{x^2}$ (2 points)

(b) Draw the triangle that is formed by the tangent line to the curve $y = x^2 - 4x + 5$ at $(1, 2)$ and the coordinate axes. What is the area of this triangle? (2 points)

Problem 9. (4 points)(a)-(c) Find $f'(x)$.

(a) $f(x) = (x^2 - 2x + 1)(x^3 - 1)$ (1 point)

(b) $f(x) = \dfrac{2\sqrt{x} - 3^{-1}}{3x^2 + 3}$ (1 point)

(c) Find all values of x at which the tangent line to $y = \dfrac{x^2 + 1}{x + 1}$ is parallel to the line $y = -x$. (2 points)

Problem 10. (4 points)

(a)-(b) Find $\dfrac{dy}{dx}$.

(a) $y = \cos x \sin x$ (1 point)

(b) $y = \sec x \cot x$ (1 point)

(c) Find an equation of the line tangent to the curve $y = \cos x$ at $x = \dfrac{\pi}{4}$. (2 points)

Problem 11. (4 points)

(a)-(b) Find $f'(x)$.

(a) $f(x) = (5x^3 - 3x + 5^{-2})^6$ (1 point)

(b) $f(x) = \sqrt{4x - 3\cos^2 x}$ (1 point)

(c) Find an equation of the tangent line to the curve $y = x \sin(2x)$ at $x = \pi$. (2 points)