Name: _____ ID: _____ Section: _____ Seat: ____

Mahidol University International College

Midterm Exam, Trimester 2, 2014-2015
ICMA/ICNS 102, ICMA 106 Principles of Mathematics, Calculus I

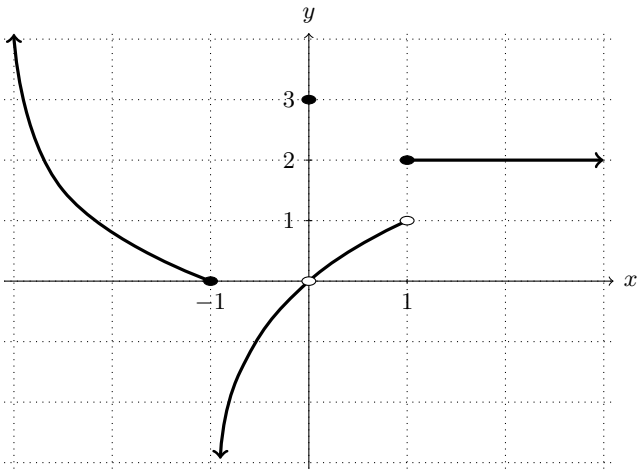
Saturday, 14 February 2015 10:00 to 11:50 60 points, 35%

Directions: Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section in the space provided on every page. The last page can be used for scratch-work.

Question 1 (Total: 6 points)

out of 6 points

For the function f given below, find the following limits. If a limit does not exist, write DNE, $+\infty$ or $-\infty$ where appropriate. (6 points)



1.1 $\lim_{x\to-\infty} f(x) =$ ____

1.2 $\lim_{x\to-1^+} f(x) =$ ____

1.3 $\lim_{x\to-1} f(x) =$ ____

1.4 $\lim_{x\to0} f(x) =$ ____

1.5 $\lim_{x\to1^-} f(x) =$ ____

1.6 $\lim_{x\to+\infty} f(x) =$ ____

Name: _____ ID: _____ Section: _____ Seat: _____

Question 2 (Total: 6 points)

out of 6 points

Find the following limits. If a limit does not exist, write DNE, $+\infty$, or $-\infty$ where appropriate. Write the final answers in the provided boxes. Use the space below for scratch work (which will not be marked).

2.1 $\lim_{x\to 1} \dfrac{3x^2-4x+4}{x+6-x^2} =$ (1 point)

2.2 $\lim_{x\to -2} \dfrac{-4x}{x+6-x^2} =$ (1 point)

2.3 $\lim_{x\to 16} \dfrac{x-16}{16(\sqrt{x}-4)} =$ (1 point)

2.4 $\lim_{x\to 2^-} \dfrac{2x}{x^2-5x+6} =$ (1 point)

2.5 $\lim_{x\to 1^-} \dfrac{x-1}{1-x} =$ (1 point)

2.6 $\lim_{x\to -\infty} \dfrac{3x^3-4x+4}{x-6+2x^2}$ (1 point)

Scratch-work

Name: _____ ID: _____ Section:_____ Seat: ____

Question 3 (Total: 6 points)

out of 6 points

3.1 Find the limit $\lim_{x\to+\infty} \sqrt{4x^2 - 2x} - 2x$ (3 points)

3.2 Use the Intermediate Value Theorem to show that the equation

$$\sqrt{3x^4 + 1} - \sqrt{x + 2} = 0$$

has a root in the interval $[0, 1]$. You must explain why you can use the intermediate value theorem in this case. Note that $\sqrt{2} \approx 1.414$ and $\sqrt{3} \approx 1.732$. (3 points)

Name: _____ ID: _____ Section: _____ Seat: ____

Question 4 (Total: 6 points)

out of 6 points

4.1 What down the three conditions for the statement: "A function $f(x)$ is continuous at $x = a$." (2 points)

4.2 Each of the following functions has a discontinuity at $x = 1$. Determine whether it is **removable, jump,** or **infinite**. Write the type of discontinuity in the provided boxes and show your work below. (4 points)

(a) $f(x) = \begin{cases} \dfrac{x+1}{x-1}, & x < 1 \\ x+1, & x \geq 1 \end{cases}$ **Ans:** f has a/an ____ discontinuity at $x = 1$.

(b) $g(x) = \begin{cases} \dfrac{2x-2}{x^2-1}, & x < 1 \\ x^2-1, & x > 1 \end{cases}$ **Ans:** g has a/an ____ discontinuity at $x = 1$.

Question 5 (Total: 7 points)

out of 7 points

5.1 Find the value of k so that the function $f(x)$ is continuous at $x = \pi$.

$$f(x) = \begin{cases} x + k\cos(x), & x \leq \pi \\ 2k\cos(x) - x + x^2\sin(x), & x > \pi \end{cases}$$

(3 points)

5.2 Find all value(s) of x at which $f(x)$ is discontinuous. Justify your answer.

$$f(x) = \begin{cases} \dfrac{x+3}{x^2 - x - 2}, & x < 1 \\ x^2 - x - 2, & x > 1 \end{cases}$$

(4 points)

Question 6 (Total: 11 points)

out of 11 points

6.1 Let $f(x) = \dfrac{x}{x+1}$.

(a) Use the definition of the derivative to find $f'(x)$. (4 points)

(b) State the quotient rule of differentiation and use it to find $f'(x)$. (3 points)

6.2 Let f be a function defined by $f(x) = 3x^2 - 4 - \frac{x^3}{2}$. Find the x-coordinates of any point where $f(x)$ has a tangent line that is parallel to the line $y = -18x - 3$. (4 points)

Question 7 (Total: 11 points)

out of 11 points

7.1 Given that $f(3) = 2$, $f'(3) = 10$, $g(3) = \frac{1}{2}$, and $g'(3) = -3$, find an equation for the tangent line to the graph of $y = f(x)g(x)$ at $x = 3$. (5 points)

7.2 During the first 50 seconds of a rocket flight, the rocket is propelled straight up so that in t seconds it reaches a height of $s = 0.4t^3$ ft.

(a) How high does the rocket travel in 5 seconds? (2 points)

(b) What is the average velocity of the rocket during the first 5 seconds? (2 points)

(c) What is the instantaneous velocity of the rocket at the end of 5 seconds? (2 points)

Question 8 (Total: 7 points)

out of 7 points

Multiple-Choice. In each of the following questions, circle the correct choice. You may use any blank space for scratch work.

8.1 Which of the following is equivalent to $\frac{d}{d\theta}(\sin\theta)\Big|_{\theta=\frac{\pi}{3}}$? (1 point)

A. $\lim\limits_{x\to\frac{\pi}{3}} \dfrac{\sin\theta - \frac{1}{2}}{\theta - \frac{\pi}{3}}$

B. $\lim\limits_{x\to\frac{\pi}{3}} \dfrac{\sin\theta - \frac{\sqrt{3}}{2}}{\theta - \frac{\pi}{3}}$

C. $\lim\limits_{x\to 0} \dfrac{\sin\theta - \frac{\sqrt{3}}{2}}{\theta - \frac{\pi}{3}}$

D. $\lim\limits_{h\to 0} \dfrac{\sin(\theta+h) - \sin\theta}{h}$

E. $\lim\limits_{h\to 0} \dfrac{\cos(\theta+h) - \cos\theta}{h}$

8.2 If $f(x) = \dfrac{x^3}{\sqrt[3]{x}}$, then which of the following is $f'(x)$? (1 point)

A. $\frac{8}{3}x\sqrt[3]{x^2}$

B. $\frac{3}{11}x\sqrt[3]{x^2}$

C. $\frac{8}{3}x^2\sqrt[3]{x^2}$

D. $3x^3$

E. $\frac{10}{3}x\sqrt[3]{x^2}$

8.3 What is $\lim\limits_{\theta\to 0} \dfrac{\frac{d}{d\theta}\cos(3\theta)}{\frac{d}{d\theta}\theta^2}$? (1 point)

A. $-\infty$

B. $-\frac{2}{3}$

C. 1

D. $-\frac{9}{2}$

E. does not exist

8.4 What is $\dfrac{d}{dx}\left(\dfrac{\cos x}{\sin x + 1}\right)$? (1 point)

A. $\sin x + 1$

B. $-\csc^2 x - \sin x$

C. $-\csc^2 x + 1$

D. $\frac{-\sin x}{1+\sin x}$

E. $\frac{-1}{1+\sin x}$

8.5 Find the equation of the line that is tangent to the curve $xy - x + y = 2$ at the point where $x = 0$. (1 point)

A. $y = -x$
B. $y = \frac{1}{2}x + 2$
C. $y = x + 2$
D. $y = 2$
E. $y = -x + 2$

8.6 If $3xy + 2y^2 = 5$, find $\dfrac{dy}{dx}$ at $(1, 1)$. (1 point)

A. $-\frac{3}{7}$
B. 0
C. $\frac{3}{7}$
D. 7
E. undefined

8.7 The slope of the line tangent to $y = \tan^2(3x)$ at $x = \frac{\pi}{12}$ is (1 point)

A. -12
B. 6
C. -6
D. 0
E. 12

Blank page

End of exam.