



## Mahidol University International College

### Midterm Exam, Trimester 3, 2014-2015 ICMA/ICNS 102, ICMA 106 Principles of Mathematics, Calculus I

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Saturday, 30 May 2015      14:00 to 15:50

70 points, 35%

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**Directions:** Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section number in the space provided on every page. The last page can be used for scratch-work.

1. Evaluate the following limits. Use the symbols  $\pm\infty$  where appropriate. Conclude your final answers in the provided spaces.

(a)  $\lim_{x \rightarrow \sqrt{5}} (9 - x^2)^{-5/2}$  (2 points)

Answer: \_\_\_\_\_

(b)  $\lim_{x \rightarrow -3^+} \frac{x^2 - 7x + 10}{x^2 + x - 6}$  (2 points)

Answer: \_\_\_\_\_

(c)  $\lim_{y \rightarrow 2} \frac{\sqrt{y} - \sqrt{2}}{y^2 - 2y}$  (2 points)

Answer: \_\_\_\_\_

(d)  $\lim_{x \rightarrow 0} \frac{1 + \pi/x}{2 + 1/x}$  (2 points)

Answer: \_\_\_\_\_

(e)  $\lim_{t \rightarrow 0} \frac{\cos^2 t - 1}{t}$  (2 points)

Answer: \_\_\_\_\_

(f)  $\lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{\sin(3x)}$  (2 points)

Answer: \_\_\_\_\_

(g)  $\lim_{x \rightarrow 0^-} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$  (2 points)

Answer: \_\_\_\_\_

2. Suppose that  $x^4 \leq f(x) \leq 2x^4$  for all  $x \in [-1, 1]$ . Find  $\lim_{x \rightarrow 0} \frac{f(x)}{x^2}$ . State clearly which theorem you have used to obtain the result. (4 points)

3. Let  $f(x) = \begin{cases} \frac{\sin(3x)}{x} & \text{for } x \neq 0 \\ 2 & \text{for } x = 0. \end{cases}$  Determine whether  $f$  is continuous at 0. (5 points)

4. (a) Complete the statement of the Intermediate Value Theorem:

If  $f$  is continuous on a closed interval  $[a, b]$  and  $k$  is any real number between  $f(a)$  and  $f(b)$ , inclusive, then

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(1 point)

(b) Show that the equation  $2x - \cos x = 1$  has at least one solution in the interval  $[0, \pi]$ .

(4 points)

5. Evaluate the following limit:

(4 points)

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{4x^2 - x} - 2x}{2}.$$

6. (a) For a function  $f$ , give the definition of its derivative  $f'(x)$  in terms of a limit. (1 point)

(b) Use the definition in the previous part to determine  $f'(x)$  where  $f(x) = \sqrt{x+5}$ . (4 points)

7. Let  $f(x) = \frac{1}{1-x}$ . Find  $f'''(2)$ . (3 points)

8. A curve has equation  $3x^2 - 4xy + 2y^2 - 6 = 0$ .

(a) Show that  $\frac{dy}{dx} = \frac{3x - 2y}{2x - 2y}$ . (3 points)

(b) Find all points on the curve where the tangent lines are parallel to the  $x$ -axis. (3 points)

9. If  $y = u^{3/2}$  and  $u = 1 + x^2$ , find  $\frac{dy}{dx}$ . (3 points)

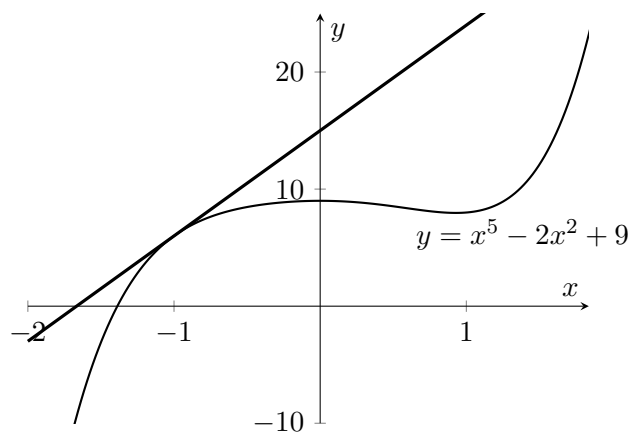
10. Find  $\frac{dy}{dx}$  if

(a)  $y = 3^e + x\sqrt{x} + \frac{3}{x^2}$  (3 points)

(b)  $y = \sec(x^2 - 2x)$  (3 points)

(c)  $y = \sqrt[3]{\frac{4x^2 + 1}{x + 2}}$  (3 points)

11. A curve has equation  $y = x^5 - 2x^2 + 9$ . The point  $P$  with coordinates  $(-1, 6)$  lies on the curve.



- (a) Find the equation of the line  $L$  which is tangent to the curve at the point  $P$ , giving your answer in the form  $y = mx + c$ . (4 points)

- (b) Find the area of the triangle formed by the line  $L$ , the  $x$ -axis, and the  $y$ -axis. (3 points)



Name: \_\_\_\_\_ ID: \_\_\_\_\_ Section: \_\_\_\_\_ Seat: \_\_\_\_\_

12. A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must air be removed when the radius is 9 cm? (5 points)

Name: ----- ID: ----- Section: ----- Seat: -----

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Scratch-paper