# Mahidol University International College

**Principles of Mathematics / Calculus I**
**ICMA102 / ICNS102 / ICMA106**
**Midterm Exam, Trimester 1, 2015-16**

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Saturday 24 October 2015 | 12:00 – 13:50 | 66 points, 30%

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Name: _____ I.D.: _____

Section: _____ Seat: _____

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**Directions:** This exam contains 10 pages and 12 questions. Show all your work clearly. A calculator is NOT allowed for this exam. Scratch-paper can be found in the last sheet which can be removed.

1. Evaluate the following limits. Use the symbols $\pm\infty$ where appropriate.

   (a) $\lim_{x\to 2\sqrt{3}} \sqrt{16-x^2}$ **[2]**

   (b) $\lim_{x\to 4^-} \dfrac{x^2+4x}{x^2-3x-4}$ **[2]**

   (c) $\lim_{y\to 1} \dfrac{\sqrt{10y-9}-1}{y-1}$ **[2]**

(d) $\lim_{x \to -4} \dfrac{1/4 + 1/x}{4 + x}$ **[2]**

(e) $\lim_{t \to 0} \dfrac{\sec t - 1}{t}$ **[2]**

(f) $\lim_{x \to 0} \dfrac{1}{\sin(2x)\cot(3x)}$ **[2]**

(g) $\lim_{x \to 0} \dfrac{\sqrt{x^2 + 8x + 25} - 5}{x}$ **[2]**

2. Determine if the following function is continuous at $x = 3$, and at $x = 1$. Justify your answers. **[5]**

$$f(x) = \begin{cases} \dfrac{x^2 - 2x - 3}{x - 3} & , x < 3 \\ 4 & , x = 3 \\ \dfrac{3x - 1}{x - 1} & , x > 3 \end{cases}$$

3. Find $\lim\limits_{x \to 0^+} \sqrt{x^3 + x^2} \sin\left(\dfrac{\pi}{x}\right)$. State clearly which theorem you have used to obtain the result. *(Hint: What are the maximum and minimum values of the sine function?)* **[4]**

4. Evaluate the following limit: **[4]**

$$\lim_{x\to+\infty} \frac{\sqrt{9x^2+x}-3x}{2}.$$

5. (a) Complete the statement of the Intermediate Value Theorem: **[1]**

If $f$ is continuous on a closed interval $[a,b]$ and $k$ is any real number between $f(a)$ and $f(b)$, inclusive, then

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(b) Show that the equation $\frac{3}{2}\tan x = \frac{x}{\pi}+1$ has at least one solution in the interval $[0,\pi/4]$. **[3]**

6. (a) For a function $f$, give the **definition** of its derivative $f'(x)$ in terms of a limit. **[1]**

(b) Use the **definition** in the previous part to find $f'(x)$ where $f(x) = \sqrt{1-x}$. **[3]**

(c) Now use **rules** of differentiation to find $f'(x)$. **[2]**

7. Let $g$ be a function such that $g'(x) = 8\sqrt{x}(1-2x^2)$. Find $g'''(1)$. **[3]**

8. Find $\dfrac{dy}{dx}$ if

(a) $y = \pi^3 + x^e + \sin(\pi^2 x)$ **[2]**

(b) $y = (7 - x)^7 + \cos(7)$ **[2]**

(c) $y = (3x + 1)^5 \cot x$ **[2]**

(d) $y = \dfrac{(1 + x)^4}{1 + x^4}$ **[2]**

(e) $y = \sqrt{\sec(\sqrt{x})}$ **[3]**

9. Let $f(x) = \dfrac{x}{1+x}$. Find all values of $x$ at which the tangent lines to the curve $y = f(x)$ are perpendicular to the line $y = -4x + 1$. **[4]**

10. Suppose that $y = \tan(u)$, $u = \sqrt{t}$, $t = x^2 + 1$. Use the Chain Rule to find $\dfrac{dy}{dx}$. Write your answer as a function of $x$ only. **[3]**

11. (a) Find $\dfrac{dy}{dx}$ if $xy = x^2 + \cos(y^2)$. **[2]**

(b) Find $\dfrac{d^2y}{dx^2}$ if $y^2 = 4x + 1$. Write your answer in terms of $x, y$ only. **[3]**

12. Find the slope of the tangent line to the following curve at the point with $y$-coordinate equal $\pi/6$. **[3]**

$$\sin y - 2x = 5y$$

(scratch-paper, write your name on top right corner if removed)

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