Mahidol University International College
Midterm Examination
ICMA/ICNS 102, ICMA 106: Principle of Mathematics, Calculus I
Second Trimester 2015-2016

Instructions. The exam consists of 9 main problems with points indicated in each problem. Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section instructor's name in the space provided on every page. Two pages of scratch-paper are provided in the back of this exam.

Name.....I.D. .....Instructor's Name.....

SCORE

Problem 1.

(a)-(e) Find the limits.

(a) $\lim_{x\to 1} \dfrac{x^2-2x+1}{x^2+2x-3}$ (1 point)

(b) $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x}-2}$ (1 point)

(c) $\lim_{x \to 9^-} \dfrac{x-3}{\sqrt{x}-9}$ (1 point)

(d) $\lim_{x \to 1^+} \dfrac{x-2}{x-1}$ (1 points)

(e) $\lim_{x \to 0} \dfrac{\cos 3x \sin 2x}{3x}$ (1 points)

Name.....I.D. .....Section.....

SCORE

Problem 2.

(a) Find the value of k for which $f(x)$ is continuous everywhere. Justify your answer.

$$f(x) = \begin{cases} 7 + k(x-2), & x < 4, \\ 2x + \dfrac{4}{x}, & x \geq 4 \end{cases}$$

(2 points)

(b) Use the Intermediate Value Theorem to show that the equation

$$x^{10} + 4x - x^9 - 2 = 0$$

has a root in the interval $[-1, 1]$. (2 points)

Name.....I.D. .....Section.....

SCORE

Problem 3.

(a) Let $f(x) = x^2 + 1$. Use the definition of the derivative in part (a) to find $f'(x)$. (2 points)

(b) Given that the tangent line to $y = g(x)$ at the point $(2, 4)$ passes through the point $(-1, 2)$, find $g'(2)$. (2 points)

Name.....I.D. .....Section.....

SCORE

Problem 4.

Find $\dfrac{dy}{dx}$.

(a) $y = (3x^5 - x^2 + 2)(x^2 - 3)$ (1 point)

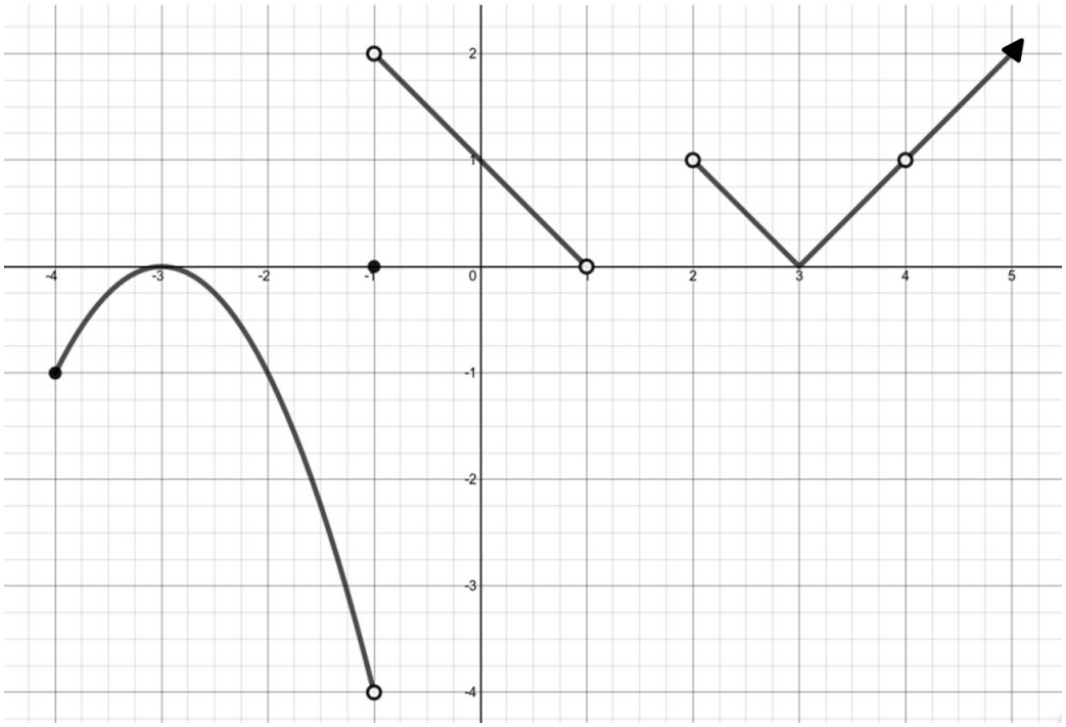
(b) $y = \dfrac{3x^5 - x^2 + 2}{x^2 - 3}$ (1 point)

Name.....I.D. .....Section.....

SCORE

Problem 5.

(2 points) Answer the following questions/fill in the blanks about the graph of the function $f(x)$ below:



(a) Identify the x-value(s) for which $f'(x) = 0$. _____

(b) Identify the x-value(s) for which $f'(x)$ does not exist:_____

(c) Draw a rough sketch of $f'(x)$ on the graph above, labeling key points.

Name.....I.D. .....Section.....

SCORE

Problem 6.

6.1 (2 points) Find $f'(x)$ of the following functions using any of the derivative properties. Indicate clearly the rule(s) you are using to get to your final answer. You do not need to simplify.

(a) $f(x) = \dfrac{1}{\sqrt[5]{1-2x}}$

(b) $f(x) = \cos(\sqrt{x}) + \sqrt{\cos x} + \sqrt{\cos(\sqrt{x})}$

6.2 (2 points) For the following, find h' in terms of f' and g':

(a) $h(x) = \dfrac{f(2x)g(x)}{f(x) - g(x)}$

(b) $h(x) = f(g(\sec(4x)))$

Name.....I.D. .....Section.....

SCORE

Problem 7.

7.1 (2 points) Find all value(s) of x for which the tangent line of $f(x) = \dfrac{(x-4)(x-2)}{x-1}$ is parallel to $y = 5 - 2x$.

7.2 (2 points) Find the equation of the tangent line to

$$y\sin(2\pi x) + x = xy + \tan^2(2-x)$$

at $(2, 1)$.

Name.....I.D. .....Section.....

SCORE

Problem 8.

(2 points) A particle moves along a vertical line so that its y-coordinate (in meters) at time t (in seconds) is $y = t^3 - 12t + 3$ for $t \geq 0$. Answer the following questions about this particle:

(a) Determine its average velocity on the interval $0 \leq t \leq 3$.

(b) Determine its velocity at $t = 2$.

(c) Determine its acceleration at $t = 2$.

(d) Is this particle speeding up or slowing down at $t = 2$? Explain.

Name.....I.D. .....Section.....

SCORE

Problem 9.

9.1 (2 points) A waterskier skis over a ramp of length 5 meters with an angle of elevation of 30 degrees from the horizontal at a speed 10 m/s. How fast is she rising as she leaves the ramp?

9.2 (1 point) Evaluate dy if $y = x^3 - 2x^2 + 1, x = 2, dx = 0.2$. Use your result to estimate the value of y when $x = 2.2$.

(Scratch-paper)

(Scratch-paper)