Mahidol University International College

Principles of Mathematics / Calculus I
ICMA102 / ICNS102 / ICMA106
Final Exam, Trimester 3, 2017-18

Saturday 21 July 2015 10:00 – 11:50 60 points, 40%

Name: _____ I.D.: _____

Section: _____ Seat: _____

Directions: This exam paper contains 10 pages and 10 questions. Show all your work clearly. A calculator is NOT allowed for this exam. Scratch-paper can be found in the last sheet, which can be removed.

1. (a) Let f be a differentiable function. Write down the formula for the local linear approximation of $f(x)$ near $x = a$. **[2]**

(b) Use local linear approximation to approximate the value of $\frac{1}{\sqrt[3]{8.1}}$. Write your final answer in the form $\frac{a}{b}$ where a, b are integers. **[4]**

2. The following questions refer to the function $f(x) = 3x^5 - 5x^3 + 3$.

   (a) Determine intervals where the function is increasing and decreasing, intervals where the graph is concave up and concave down, points (x, y) of all relative extrema and (only) the x-coordinates of all inflection points. Show your work below and write your final answers in the space provided. **[6]**

Intervals of increasing = _____

Intervals of decreasing = _____

Relative maximum point(s) = _____

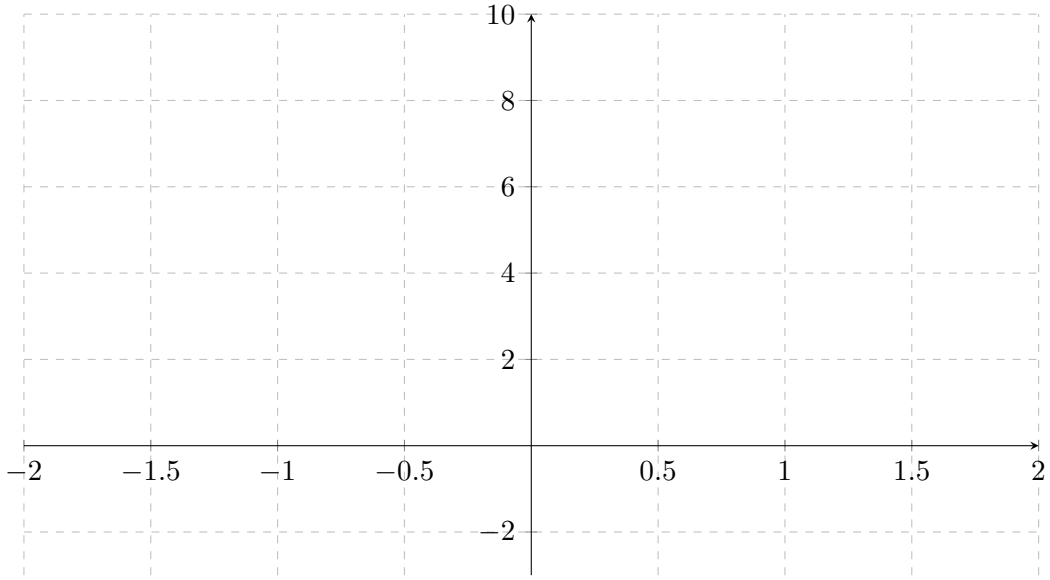
Relative minimum point(s) = _____

Intervals of concave up = _____

Intervals of concave down = _____

Inflection points (only values of x) = _____

(b) Sketch the graph of this function. Label relative extrema points on the graph. Concavity of the graph should be correct. **[3]**



(c) How many solutions does the equation $f(x) = 0$ have? **[1]**

3. The height of a triangle is increasing at a rate of 3 cm per minute while the base of the triangle is decreasing at a rate of 2 cm per minute. At the instant when the height is 8 cm and the base is 4 cm, what is the rate of change of the area of the triangle? **[5]**

4. An open box is to be made from an 8-inch by 15-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the extreme volume? Is this volume maximum or minimum? Justify your answer. **[5]**

5. Find the absolute extrema of the function $f(x) = \dfrac{5x}{1+x^2}$ on the interval $[0.5, 10]$. **[5]**

6. Evaluate the following integrals.

(a) $\int \frac{2x + 3x^2}{3\sqrt{x}}\, dx$ **[3]**

(b) $\int 3\sec x(2\sec x - 3\tan x)\, dx$ **[3]**

(c) $\int \frac{2x^2}{(3x^3 + 5)^5}\, dx$ **[4]**

7. Let $f(x) = \int_{-2}^{x} (t-1)(t+1)^9 \, dt$. Find

   (a) $f(-2)$ **[2]**

   (b) $f'(-2)$ **[2]**

   (c) $f''(-2)$ **[3]**

8. Determine the following integral: $\int_0^1 x^2 + e^2 \, dx$. **[2]**

9. Estimate the area under the graph of $f(x) = 1/x$ over the interval $[2, 4]$ using *three* subintervals and *right* endpoints. Write your final answer in the simplest form. **[5]**

10. Find the area of the region bounded by the graphs of $y = x^2 - 1$, $y = x + 1$, $x = 0$ and $x = 3$ **[5]**

(scratch-paper)