Name ..... I.D. ..... Section...

Mahidol University International College
Final Examination
ICMA106 Calculus I
First Trimester 2019–2020
7 December 2019, 12:00 - 13:50

Instructions. The exam consists of 9 main problems (**95 points=35%**) with points indicated in each problem. Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section instructor's name in the space provided on the first page. If not otherwise specified, your answer to every problem must be in **simplest** form.

SCORE

Problem 1. (10 points)

Let $f(x) = (\frac{1}{2}x + \cos x)^2$.

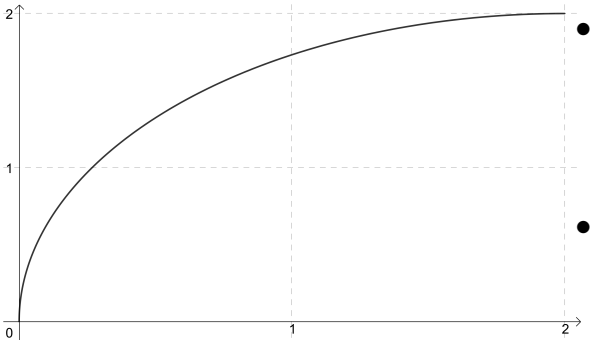
(a) Find $f'(x)$ and $f'(\frac{\pi}{6})$. (5 points)

(b) Using the Second Derivative Test, or otherwise, determine whether $f(x)$ has a local minimum, or local maximum at $x = \frac{\pi}{6}$. (No point is awarded if no justification is given.) (5 points)

SCORE

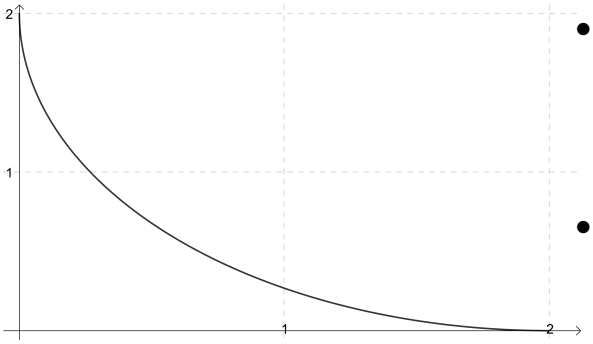
Problem 2. (10 points)

(a) Below is the graph of $y = f(x)$ over the interval $[0, 2]$. Determine the signs of $f'(1)$ and $f''(1)$ (circle the correct option). (2 points)



- $f'(1)$ is POSITIVE / NEGATIVE.
- $f''(1)$ is POSITIVE / NEGATIVE.

(b) Below is the graph of $y = g(x)$ over the interval $[0, 2]$. Determine the signs of $g'(1)$ and $g''(1)$ (circle the correct option). (2 points)



- $g'(1)$ is POSITIVE / NEGATIVE.
- $g''(1)$ is POSITIVE / NEGATIVE.

(c) Let $h(x)$ be a function such that $h'(x) = 3+2\sin(x)$ and $h(\pi) = 3\pi$. Determine the value of $h(2\pi)$. (6 points)

SCORE

Problem 3. (10 points)

Let $f(x) = -x^3 - 2x^2 + 4x + 8$. Sketch a graph of this polynomial and label the coordinates of the intercepts, relative extrema, and inflection points.
Hint: $-x^3 - 2x^2 + 4x + 8 = -(x+2)^2(x-2)$

SCORE

Problem 4. (10 points)

Find the absolute maximum and minimum values of $f(x) = x^3 - 3x^2 + 1$ on the interval $[-\frac{1}{2}, 4]$.

SCORE

Problem 5. (10 points)

A farmer plans to fence a rectangular pasture by the side of a river. The pasture must contain $5,000$ square meters to provide enough grass for the herd. What dimensions will require the least amount of fencing if no fencing is needed along the river?

SCORE

Problem 6. (10 points)

a) (4 points) **Riemann sum**. Use $n = 4$ rectangles and right endpoints to sketch and approximate the area of the region bounded by $f(x) = 2x + 5, 0 \leq x \leq 2$ and the $x-$axis.

b) (3 points) Let $F(x) = \int_0^x \cos t \, dt$. Evaluate $F(0), F\left(\frac{\pi}{6}\right)$, and $F\left(\frac{\pi}{2}\right)$.

c) (3 points) Let $G(x) = \int_0^x \sec t \, dt$. Evaluate $G(0)$ and $G'(x)$.

SCORE

Problem 7. (15 points)

Evaluate the integrals (a - e).

a) $\displaystyle\int \frac{z^3 - z}{(z^2 - 1)^3}\, dz$

b) $\displaystyle\int (t^4 + 1)(t^3 + 1)\, dt$

c) $\displaystyle\int_0^4 \sqrt{x}(x - 1)\, dx$

d) $\displaystyle\int_0^{\sqrt{\frac{\pi}{4}}} x \sec^2(x^2)\, dx$

e) $\displaystyle\int \frac{\sin x}{\sin^2 x - 1}\, dx$

SCORE

Problem 8. (10 points)

Sketch the region enclosed by the following curves and find its area:

$$x = 1 - y^2 \text{ and } x = y^2 - 1.$$

SCORE

Problem 9. (10 points)

Sketch the region enclosed by the following curves and find its area:

$$y = 5x^2 + 2, x = 0, x = 2, \text{ and } y = 0.$$

You may tear this page apart as a scratch paper.

You may tear this page apart as a scratch paper.