Name . . . . . I.D. . . . . Section. . .

Mahidol University International College
ICMA106 Calculus I Final Examination
Trimester 2/2019–2020
4 April 2020, 08:00 - 23:59

Instructions

Before you work on this exam, please read this page very carefully and sign where indicated. **Failure to follow the instructions will result in your work not being graded.**

1. This examination paper will be posted online via Canvas on **Saturday 4th April 2020 at 08:00 AM (BKK time)**. The deadline for submission is **Saturday 4th April 2020 at 11:59 PM (BKK time)**. Submission is via Canvas only. Late submission is not accepted.

2. Write your name, student ID and section clearly on the top of this page and the next page. **A submitted work without any identity will not be graded.**

3. When working on the questions, you may

   (a) download this pdf file, show your work on the file (annotation), then save/export your work into a **single pdf file** and submit it via Canvas, or

   (b) download and print this pdf file, show your work on the printed paper, then scan your work into a **single pdf file** and submit it via Canvas, or

   (c) show your work on blank A4 papers, then scan your work into a **single pdf file** and submit it via Canvas. In this case, write Item 8 with your handwriting and sign.

4. Show all your work with your handwriting only (no typing allowed). We will compare your handwriting with your previously submitted works.

5. This is a closed book examination. You are **not allowed** to use a calculator (this includes any form of mathematical software).

6. You are **not allowed** to get help or discuss any part of the exam with anyone else.

7. **Students found cheating during the examination will be penalized according to the university regulation.**

8. I hereby acknowledge that I have read, understood and agreed to the instructions of this examination paper.

Signature: . . . . . .

(Your work will not be graded, if you did not sign the above.)

Name ..... I.D. ..... Section...

Instructions. The exam consists of 9 main problems (**95 points=35%**) with points indicated in each problem. Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section instructor's name in the space provided on the first page. If not otherwise specified, your answer to every problem must be in **simplest** form.

SCORE

Problem 1. (10 points)

Let $f(x) = 2\sin x + \sin(2x)$ on $[0, 2\pi]$.

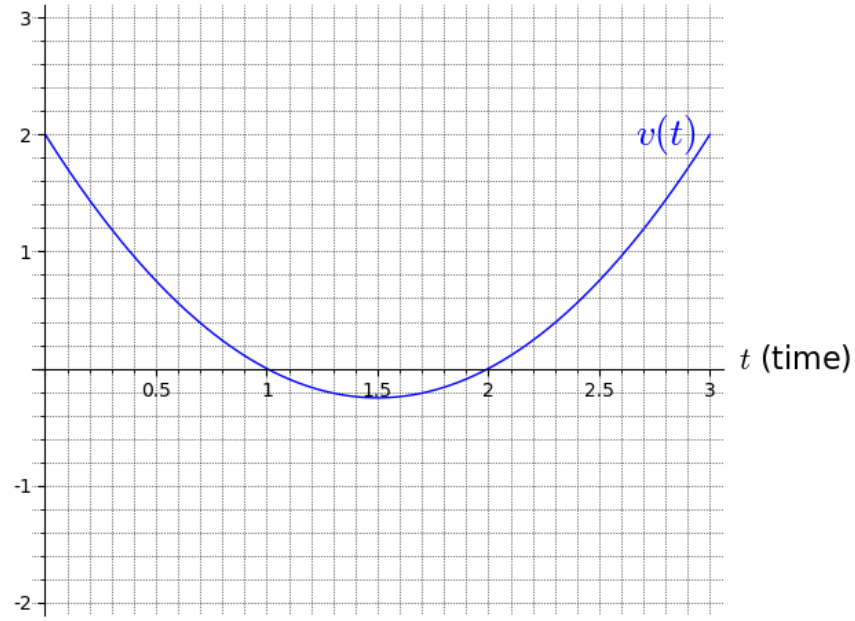
(a) Find $f'(x)$ and $f'(\frac{\pi}{6})$. (5 points)

(b) Use the Second Derivative Test, when applicable, to determine the relative extremum of the $f(x)$ at $x = \frac{\pi}{3}$. (No point is awarded if no justification is given.) (5 points)

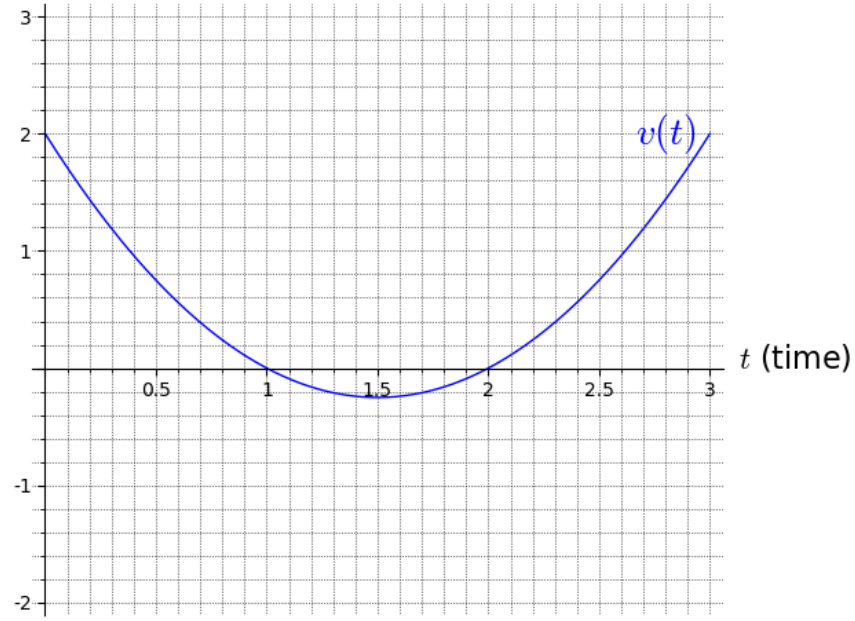
SCORE

Problem 2. (10 points)

(a) The graph of the velocity function $v(t)$ (meters/second) for a particle that moves on a horizontal line is given in the figure below. Sketch a graph of the position function $s(t)$ with this velocity $v(t) = \dfrac{ds}{dt}$. Assume that $s(0) = 0$. You may plot $s(t)$ on the same axes. (2 points)



(b) From problem (a), sketch a graph of the acceleration $a(t) = \dfrac{dv}{dt}$. You may plot $a(t)$ on the same axes. . (2 points)



(c) Find a function $f(x)$ such that $\dfrac{df}{dx} = 1 - 2x + \sin x$ and $f(\pi) = \pi$. (6 points)

SCORE

Problem 3. (10 points)

Let $f(x) = x^3 - 9x^2 + 24x - 16$. Sketch a graph of this polynomial and label the coordinates of the intercepts, relative extrema, and inflection points.
Hint: $x^3 - 9x^2 + 24x - 16 = (x - 1)(x - 4)^2$

SCORE

Problem 4. (10 points)

Find the absolute maximum and minimum values of $f(x) = (x+5)\sqrt{x-1}$ on the interval $[1, 4]$.

SCORE

Problem 5. (10 points)

A person would like to cut a 2-meter-long piece of wire into two pieces. One piece will be bent into the shape of a circle and the other into the shape of a square. How should the wire be cut so that the sum of the areas is maximum?

SCORE

Problem 6. (10 points)

a) (6 points) **Riemann sum**. Use $n = 4$ rectangles and left endpoints to sketch and approximate the area of the region bounded by $f(x) = 20 - 3x, 3 \leq x \leq 5$ and the $x-$axis.

b) (4 points) Let $F(x) = \int_0^x \sec^2 t \, dt$. Evaluate $F(0), F\left(\frac{\pi}{4}\right)$, and $F''(x)$.

SCORE

Problem 7. (12 points)

Evaluate the integrals (a - d).

a) $\int (2\sqrt{x} - 1)(3\sqrt{x} + 2)\, dx$

b) $\int_{-2}^{-1} (5 - 2x - x^2)\, dx$

c) $\int 5x(1-2x^2)^{3/2}\,dx$

d) $\int_{\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{2}}} 8x\csc(x^2)\cot(x^2)\,dx$

SCORE

Problem 8. (13 points)

a) (6 points) Evaluate $\displaystyle\int_0^5 f(x)\,dx$ where $f(x) = \begin{cases} 3x-1, & x \leq 2 \\ 9-x, & x > 2. \end{cases}$

b) (7 points) Find the **total area** between the graph of $y = x^2 - 4$ and the x-axis on the interval $[0, 3]$.

SCORE

Problem 9. (10 points)

Sketch the region enclosed by the following curves and find its area:

$$y = \sin(x), \quad y = \cos(x), \quad x = 0, \quad x = \pi/4.$$

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