



Mahidol University International College

Midterm Exam, Trimester 1, 2017-2018 ICMA/ICNS 102, ICMA 106 Principles of Mathematics, Calculus I

Saturday, 21 October 2017 10:00 to 11:50

76 points, 35%

Directions: Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section number in the space provided on every page. The last page can be used for scratch-work.

1. Evaluate the following limits. Use the symbols $\pm\infty$ where appropriate. Conclude your final answers in the provided spaces.

(a) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 5x + 6}$ (3 points)

Answer: _____

(b) $\lim_{s \rightarrow 3^-} \frac{-s}{3 - s}$ (3 points)

Answer: _____

(c) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{4 - x}$ (3 points)

Answer: _____

(d) $\lim_{x \rightarrow 0} \frac{\sin(6x)}{2x}$ (3 points)

Answer: _____

(e) $\lim_{\theta \rightarrow 0} \frac{\theta - 3 \cos \theta}{\theta - 1}$ (3 points)

Answer: _____

(f) $\lim_{x \rightarrow -\infty} \frac{-3x^4 - (5x - 7)^3 - x}{-3x + 2x^4 + 1}$ (3 points)

Answer: _____

2. Suppose that $4x - 9 \leq f(x) \leq x^2 - 4x + 7$ for $x \geq 0$. Find $\lim_{x \rightarrow 4} f(x)$. State clearly which theorem you have used to obtain the result. (4 points)

3. Let $f(x) = \begin{cases} \frac{4x - 4 \cos x}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0. \end{cases}$ Determine whether f is continuous at 0. (4 points)

4. (a) For a function f , give the definition of its derivative $f'(x)$ in terms of a limit. (1 point)

(b) Use the definition in the previous part to determine $f'(x)$ where $f(x) = x^2 - 3$. (5 points)

5. Let $f(x) = \frac{1}{2-x}$. Find $f''(1)$. (4 points)

6. A curve has equation $x^2 + y^2 = 25$.

(a) Find $\frac{dy}{dx}$. (4 points)

(b) Find the equation of the tangent line to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$. (4 points)

7. If $y = \sin u$ and $u = x - x^3$, find $\frac{dy}{dx}$. (4 points)

8. Find $\frac{dy}{dx}$ if

(a) $y = \frac{1}{\pi} - x\sqrt{x^3} + \frac{x^3 - \sqrt{x}}{x\sqrt{x}}$ (4 points)

(b) $y = \csc(x^4 - 2x)$ (4 points)

(c) $y = \sqrt[3]{\frac{x^3 + 1}{x + 1}}$ (4 points)

(d) $y = \sec(\cot(1 - x^3))$ (4 points)

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9. Air is being pumped into a spherical balloon so that its volume increases at a rate of $100 \text{ cm}^3/\text{s}$.
How fast is the radius of the balloon increasing when the diameter is 50 cm ? (6 points)

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10. [Bonus!] Car A is traveling west at 50 mi/h and car B is traveling north at 60 mi/h . Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection? ($\text{mi} = \text{mile}, h = \text{hour}$)
(6 points)

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