



## Mahidol University International College

### Midterm Exam, Trimester 2, 2017-2018 ICMA/ICNS 102, ICMA 106 Principles of Mathematics, Calculus I

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Saturday, 17 February 2018      10:00 to 11:50

76 points, 35%

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**Directions:** Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section number in the space provided on every page. The last page can be used for scratch-work.

1. Evaluate the following limits. Use the symbols  $\pm\infty$  where appropriate. Conclude your final answers in the provided spaces.

(a)  $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 - x - 2}$  (3 points)

Answer: \_\_\_\_\_

(b)  $\lim_{t \rightarrow -3^+} \frac{t + 2}{t + 3}$  (3 points)

Answer: \_\_\_\_\_

(c)  $\lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$  (3 points)

Answer: \_\_\_\_\_

(d)  $\lim_{x \rightarrow 0} \frac{\sin(2x)}{5x}$

(3 points)

**Answer:** \_\_\_\_\_

(e)  $\lim_{\theta \rightarrow 0} \frac{3\theta - \cos \theta}{3 - \theta}$

(3 points)

**Answer:** \_\_\_\_\_

(f)  $\lim_{x \rightarrow +\infty} \frac{(5x - 7)^3 - 4x^4 - x}{3x - 2x^4 - 100}$

(3 points)

**Answer:** \_\_\_\_\_

2. Suppose that  $4x - 12 \leq f(x) \leq x^2 - 4x + 4$  for  $x \geq 0$ . Find  $\lim_{x \rightarrow 4} f(x)$ . State clearly which theorem you have used to obtain the result. (4 points)

3. Let  $f(x) = \begin{cases} \frac{2 \cos x - 2x}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0. \end{cases}$  Determine whether  $f$  is continuous at 0. (4 points)

4. (a) For a function  $f$ , give the definition of its derivative  $f'(x)$  in terms of a limit. (1 point)

(b) Use the definition in the previous part to determine  $f'(x)$  where  $f(x) = x^2 + 5$ . (5 points)

5. Let  $f(x) = \frac{1}{3-x}$ . Find  $f''(2)$ . (4 points)

6. A curve has equation  $x^2 + y^2 = 4$ .

(a) Find  $\frac{dy}{dx}$ .

(4 points)

(b) Find the equation of the tangent line to the circle  $x^2 + y^2 = 4$  at the point  $(-\sqrt{2}, \sqrt{2})$ .  
(4 points)

7. If  $y = \cos u$  and  $u = x^4 - 2x$ , find  $\frac{dy}{dx}$ .

(4 points)

8. Find  $\frac{dy}{dx}$  if

(a)  $y = \pi^2 - x^2\sqrt{x} + \frac{x^2 - \sqrt{x^3}}{x\sqrt{x}}$  (4 points)

(b)  $y = \tan(x^3 - 5x)$  (4 points)

(c)  $y = \sqrt[5]{\frac{x^2 - 1}{x + 1}}$  (4 points)

(d)  $y = \cot(\csc(1 - x^2))$  (4 points)

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9. Each side of a square is increasing at a rate of  $6 \text{ cm/s}$ . At what rate is the area of the square increasing when the area of the square is  $16 \text{ cm}^2$ ? (6 points)

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10. [Bonus!] At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM? (6 points)



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Scratch-paper

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