Mahidol University International College

Principles of Mathematics / Calculus I

ICMA102 / ICNS102 / ICMA106

Midterm Exam, Trimester 3, 2017-18

Saturday 2 June 2018	10:00 – 11:50	45 points, 30%

Name: ______________________ I.D.: ______________

Section: __________ Seat: __________

Directions: This exam contains 8 pages and 10 questions. Show all your work clearly. A calculator is NOT allowed for this exam.

1. Evaluate the following limits. Use the symbols $\pm\infty$ where appropriate.

   (a) $\lim\limits_{x\to 3\sqrt{3}} \sqrt{28 - x^2}$ **[2]**

   (b) $\lim\limits_{x\to 5^-} \dfrac{4.9999 - x}{x^2 - 4x - 5}$ **[2]**

(c) $\lim_{t \to 0} \frac{1 - \sqrt{10t + 9}}{t - 1}$ **[2]**

(d) $\lim_{x \to 0} \frac{\tan(2x)}{\tan(3x)}$ **[2]**

(e) $\lim_{x \to +\infty} \sqrt{x^2 + 8x + 1} - x$ **[2]**

2. Determine if the following function is continuous at $x = 5$. Justify your answer. **[4]**

$$f(x) = \begin{cases} 2x - 6 & , x < 5 \\ 4 & , x = 5 \\ \dfrac{3x-1}{x-3} & , x > 5 \end{cases}$$

3. Assume that $\lim_{x \to 2} f(x) = 5$ and $\lim_{x \to 2} g(x) = 3$. Evaluate the following limits: **[4]**

(a) $\lim_{x \to 2} (f(x) - 3g(x))$

(b) $\lim_{x \to 2} \dfrac{2x - g(x)}{xf(x)}$

4. (a) For a function f, give the **definition** of its derivative $f'(x)$ in terms of a limit. **[1]**

(b) Use the **definition** in the previous part to find $f'(x)$ where $f(x) = x^2 - x$. **[3]**

(c) Now use **rules** of differentiation to find $f'(x)$. **[2]**

5. Find $\dfrac{dy}{dx}$ if

   (a) $y = x^2 + e^2x + e^2$ **[2]**

   (b) $y = \sqrt{2 + \cos x}$ **[2]**

6. Let $y = x^2 - x + 1$. Find the average rate of change of y with respect to x over the interval $[-1, 2]$. **[2]**

7. Let f be a function such that $f'(x) = \dfrac{1-2x^2}{\sqrt{x}}$. Find $f'''(1)$. **[3]**

8. Find an equation of the tangent line to the curve $y = \dfrac{2}{3-x}$ at $x = 1$. **[4]**

9. Find $\dfrac{dy}{dx}$ if $5x = x^2 + \cos(y^2)$. **[4]**

10. Find an equation of the tangent line to the curve $xy^3 = 2y^3 + x$ at $x = 1$. **[4]**

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