



Mahidol University International College

Midterm Exam, Trimester 1, 2018-2019 ICMA/ICNS 102, ICMA 106 Principles of Mathematics, Calculus I

Saturday, 20 October 2018 10:00 to 11:50

76 points, 35%

Directions: Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section number in the space provided on every page. The last page can be used for scratch-work.

1. Evaluate the following limits. Use the symbols $\pm\infty$ where appropriate. Conclude your final answers in the provided spaces.

(a) $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 4x + 3}$ (3 points)

Answer: _____

(b) $\lim_{t \rightarrow -2^-} \frac{t + 3}{t + 2}$ (3 points)

Answer: _____

(c) $\lim_{t \rightarrow 1} \frac{\sqrt{t^2 + 3} - 2}{t^2 - 1}$ (3 points)

Answer: _____

(d) $\lim_{\theta \rightarrow 0} \frac{2 - 2 \cos \theta}{2\theta}$ (3 points)

Answer: _____

(e) $\lim_{x \rightarrow 0} \frac{\sin(-3x)}{4x}$ (3 points)

Answer: _____

(f) $\lim_{x \rightarrow +\infty} \frac{5x^4 - (3x + 2)^3 + 100}{5x - 2x^4 - 7}$ (3 points)

Answer: _____

2. Suppose that $1 - x^2 \leq f(x) \leq x^2 + 1$ for $-1 < x < 1$. Find $\lim_{x \rightarrow 0} f(x)$. State clearly which theorem you have used to obtain the result. (4 points)

3. Let $f(x) = \begin{cases} \frac{2 \sin x}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0. \end{cases}$ Determine whether f is continuous at 0. (4 points)

4. (a) For a function f , give the definition of its derivative $f'(x)$ in terms of a limit. (1 point)

(b) Use the definition in the previous part to determine $f'(x)$ where $f(x) = x^2 - 2$. (5 points)

5. Let $f(x) = \frac{1}{x+2}$. Find $f''(1)$. (4 points)

6. A curve has equation $y^2 = x^3 + 1$.

(a) Find $\frac{dy}{dx}$. (4 points)

(b) Find the equation of the tangent line to the curve $y^2 = x^3 + 1$ at the point $(2, 3)$. (4 points)

7. If $y = \tan u$ and $u = x^3 + 1$, find $\frac{dy}{dx}$. (4 points)

8. Find $\frac{dy}{dx}$ if

(a) $y = \sqrt{\pi} - x\sqrt{x} + \frac{x^2 + \sqrt{x^5}}{x\sqrt{x}}$ (4 points)

(b) $y = \sec(x^2 + x)$ (4 points)

(c) $y = \sqrt[3]{\frac{x^2 - 4}{x - 2}}$ (4 points)

(d) $y = \sin(\sec(1 - x^2))$ (4 points)

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9. Each side of a cube is increasing at a rate of 1 cm/s . At what rate is the volume of the cube increasing when the volume of the cube is 8 cm^3 ? (6 points)

10. [Bonus!] If $\frac{d}{dx}f(x) = g(x)$ and $\frac{d}{dx}g(x) = f(x^2)$, then $\frac{d^2}{dx^2}f(x^3) = ?$ (show your work!) (6 points)

(A) $f(x^6)$

(B) $g(x^3)$

(C) $3x^2g(x^3)$

(D) $9x^4f(x^6) + 6xg(x^3)$

(E) $f(x^6) + g(x^3)$

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