



## Mahidol University International College

### Midterm Exam, Trimester 3, 2018-2019 ICMA 106 Calculus I

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Saturday, 1 June 2019      10:00 to 11:50

76 points, 35%

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**Directions:** Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section number in the space provided on every page. The last page can be used for scratch-work.

1. Evaluate the following limits. Use the symbols  $\pm\infty$  where appropriate. Conclude your final answers in the provided spaces.

(a)  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 2x - 3}$  (3 points)

Answer: \_\_\_\_\_

(b)  $\lim_{s \rightarrow 2^+} \frac{s^2 - 9}{s - 2}$  (3 points)

Answer: \_\_\_\_\_

(c)  $\lim_{y \rightarrow 4} \frac{\sqrt{y+5} - 3}{y^2 - 16}$  (3 points)

Answer: \_\_\_\_\_

(d)  $\lim_{\theta \rightarrow 0} \frac{2 - 2 \cos \theta}{2 - \theta}$  (3 points)

**Answer:** \_\_\_\_\_

(e)  $\lim_{x \rightarrow 0} \frac{\sin(2x)}{3x}$  (3 points)

**Answer:** \_\_\_\_\_

(f)  $\lim_{x \rightarrow +\infty} \frac{5x - (3x + 2)^2 + 100}{5x^2 - 2x - 7}$  (3 points)

**Answer:** \_\_\_\_\_

2. Let  $f(x) = \begin{cases} \frac{\cos x - 1}{x} & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases}$ . Determine whether  $f$  is continuous at 0. (6 points)

3. (a) For a function  $f$ , give the definition of its derivative  $f'(x)$  in terms of a limit. (1 point)

(b) Use the definition in the previous part to determine  $f'(x)$  where  $f(x) = x^2 + 3$ . (5 points)

4. Let  $f(x) = \frac{1}{x-2}$ . Find  $f''(0)$ . (4 points)

5. Compute derivatives of the following functions.

(a)  $V(r) = (r^3 - 2r - 5)(2r + 1)$  (4 points)

(b)  $g(x) = \frac{x}{x + \frac{2}{x}}$  (4 points)

6. A curve has equation  $y^2 = x^3 + 8$ .

(a) Find  $\frac{dy}{dx}$ . (4 points)

(b) Find the equation of the tangent line to the curve  $y^2 = x^3 + 8$  at the point  $(1, 3)$ . (4 points)

7. If  $y = u^3 + 1$  and  $u = \sec x$ , find  $\frac{dy}{dx}$ . (4 points)

8. Find  $\frac{dy}{dx}$  if

(a)  $y = \sin\left(\frac{\pi}{5}\right) + x^2\sqrt{x} - \frac{x^5 + \sqrt{x^3}}{x\sqrt{x}}$  (4 points)

(b)  $y = \tan(x^3 - 1)$  (4 points)

(c)  $y = \frac{x^2 - 3}{x - 3}$  (4 points)

(d)  $y = \cot^2(x^3 - 1)$  (4 points)

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9. [Bonus!] Suppose  $f(x)$  is a twice differentiable function satisfying  $f(x^2) = f(x) - x^2$ . What is  $f'(1)$ ? (6 points)



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