



Mahidol University International College

Midterm Exam, Trimester 1, 2019-2020 ICMA 106 Calculus I

Saturday, 19 October 2019 12:00 to 13:50

76 points, 35%

Directions: Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section number in the space provided on every page. The last page can be used for scratch-work.

1. Evaluate the following limits. Use the symbols $\pm\infty$ where appropriate. Conclude your final answers in the provided spaces.

(a) $\lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x^2 - 2x - 3}$ (3 points)

Answer: _____

(b) $\lim_{s \rightarrow 2^+} \frac{s^2 - 1}{2 - s}$ (3 points)

Answer: _____

(c) $\lim_{t \rightarrow 2} \frac{\sqrt{5 + t^2} - 3}{t^2 - 4}$ (3 points)

Answer: _____

(d) $\lim_{\theta \rightarrow 0} \frac{2 - 2 \cos^2 \theta}{2\theta^2}$ (3 points)

Answer: _____

(e) $\lim_{y \rightarrow 0} \frac{\sin(-5y)}{4y}$ (3 points)

Answer: _____

(f) $\lim_{x \rightarrow -\infty} \frac{5x^4 - (3x^3 + 2)^2 + 100}{5x - 2x^4 - 7}$ (3 points)

Answer: _____

2. Suppose that $\cos^2 x - 1 \leq f(x) \leq \sin^2 x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Find $\lim_{x \rightarrow 0} f(x)$. State clearly which theorem you have used to obtain the result. (4 points)

3. Let $f(x) = \begin{cases} \frac{x^2 + x - 2}{x^2 - 1} & \text{for } x \neq 1 \\ 2 & \text{for } x = 1. \end{cases}$ Determine whether f is continuous at 1. (4 points)

4. (a) For a function f , give the definition of its derivative $f'(x)$ in terms of a limit. (1 point)

(b) Use the definition in the previous part to determine $f'(x)$ where $f(x) = x^2 + 5$. (5 points)

5. Let $f(x) = \frac{1}{2-x}$. Find $f''(1)$. (4 points)

6. A curve has equation $y^2 + 3 \cos(y) + 2y = x^2 - 1$.

(a) Find $\frac{dy}{dx}$. (4 points)

(b) Find the equation of the tangent line to the curve $y^2 + 3 \cos(y) + 2y = x^2 - 1$ at the point $(-2, 0)$. (4 points)

7. If $u = \sec x$ and $x = 3t^2 + 1$, find $\frac{du}{dt}$. (4 points)

8. Find $\frac{dy}{dx}$ if

(a) $y = e^2 - x\sqrt{x} + \frac{x^2 + \pi^2}{\sin(x)}$ (4 points)

(b) $y = \tan(1 - x^2)$ (4 points)

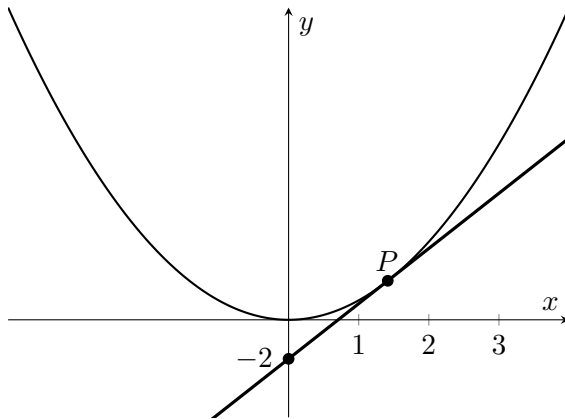
(c) $y = \left(\frac{x^3 - 8}{2x - 1}\right)^6$ (4 points)

(d) $y = \cos(\sqrt{1 - x^2})$ (4 points)

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9. A cone with radius r and height h has a volume given by $V = \frac{1}{3}\pi r^2 h$. Suppose the radius is decreasing at the rate of 1 cm/s at the height is increasing at the rate of 2 cm/s. At the moment when the radius is 10 cm and the height is 2 cm, determine the rate of change of the volume. Is the volume increasing or decreasing at that moment? (6 points)

10. [Bonus!] The line L has a positive slope and passes through the point $(0, -2)$. Suppose L is tangent to the parabola $y = x^2$. Let P be the point on the parabola that the line L also passes through. Find the coordinates of P . (6 points)



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