

Mahidol University International College

Midterm Exam, Trimester 2, 2019-2020 ICMA 106 Calculus I

Saturday, 15 February 2020

12:00 to 13:50

76 points, 35%

Directions: Show all your work clearly. A calculator is NOT allowed for this exam. Make sure to fill in your name, student I.D., and your section number in the space provided on every page. The last page can be used for scratch-work.

1. Evaluate the following limits. Use the symbols $\pm\infty$ where appropriate. Conclude your final answers in the provided spaces.

(a) $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{2x^2 + 2x - 4}$ (3 points)

Answer: _____

(b) $\lim_{x \rightarrow -1^+} \frac{x + 3}{2x + 2}$ (3 points)

Answer: _____

(c) $\lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 9t + 9} - 3}{3t}$ (3 points)

Answer: _____

(d) $\lim_{\theta \rightarrow 0} \cos\left(\frac{\pi - \pi \cos \theta}{\theta}\right)$ (3 points)

Answer: _____

(e) $\lim_{x \rightarrow 0} \frac{\sin(7x)}{-3x}$ (3 points)

Answer: _____

(f) $\lim_{x \rightarrow +\infty} \frac{2x^4 - (3x^3 + 2)^2 + 100}{3x^2 - 7x^6 - 1}$ (3 points)

Answer: _____

2. Find all values of k so that the following function is continuous at $x = 0$. (4 points)

$$f(x) = \begin{cases} \frac{\sin(kx)}{x}, & x < 0 \\ 2x + 3k^2, & x \geq 0 \end{cases}$$

3. Let $f(x) = \begin{cases} \frac{x^2 + 2x - 3}{x^2 - 1} & \text{for } x < 1 \\ 3 & \text{for } x = 1 \\ 4 - 2x & \text{for } x > 1. \end{cases}$ Determine whether f is continuous at $x = 1$. (4 points)

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4. (a) For a function f , give the definition of its derivative $f'(x)$ in terms of a limit. (1 point)

(b) Use the definition in the previous part to determine $f'(x)$ where $f(x) = x^2 - 3x$. (5 points)

5. Let $f(x) = \sec(x)$. Find the value of $f''(\pi/3)$. Simplify your answer. (4 points)

6. A curve has equation $x^3 + y^3 = 9$.

(a) Find $\frac{dy}{dx}$. (4 points)

(b) Find the equation of the tangent line to the curve $x^3 + y^3 = 9$ at the point $(1, 2)$. (4 points)

7. If $u = \tan x$ and $x = 2t^3 - t$, find $\frac{du}{dt}$. (4 points)

8. Find $\frac{dy}{dx}$ if

(a) $y = \pi^5 - \sqrt[3]{x^2} + \frac{\cos x + e^2}{x^2}$ (4 points)

(b) $y = \csc(x^3 - 3)$ (4 points)

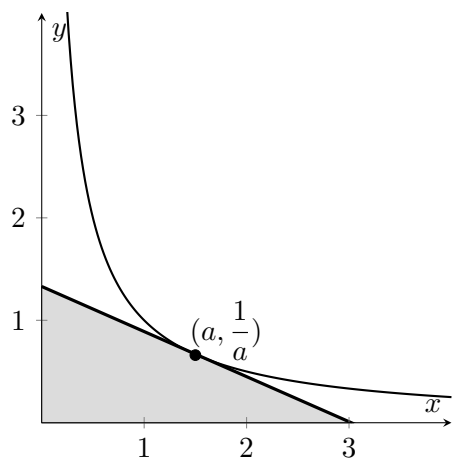
(c) $y = \left(\frac{x^3 - 8}{x - 2}\right)^4$ (4 points)

(d) $y = \sec(1 - \sin^2 x)$ (4 points)

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9. Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of $6 \text{ mi}^2/h$. How fast is the radius of the spill increasing when the area is 9 mi^2 ?
($\text{mi} = \text{mile}$) (6 points)

10. [Bonus!] Suppose $a > 0$, and consider the line L tangent to $y = \frac{1}{x}$ at $x = a$. Find the area of the triangular region between L , the x -axis, and the y -axis. (6 points)



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