

Mahidol University International College**ICNS 103 Fundamental Mathematics****Final Examination****8 January 2014 – 08.00 - 09.50****35 points**

Directions: Solve the following problems using the bottom of each page for scratch work. Write up your solution and answer (in simplified form) in the space provided. Calculators are NOT allowed for this examination.

SCORE

Problem 1: 10 points

1.1 Consider the multi-variable function:

$$f(u, v, w) = \sqrt{\ln(1 - u) + v + w^2} + w(1 - u)^2.$$

Determine $\frac{\partial f}{\partial u}$ at $(1 - e, 2, 1)$. Please simplify your answer to ONE fraction. You do not need to approximate the number e (just write it as e). (3 pts.)

1.2 For two products x and y , the joint-cost function is

$$c(x, y) = 2x\sqrt{x + y} + 6000.$$

What is the marginal cost with respect to x when 70 units of x and 74 units of y are being made? (2 pts.)

1.3 Find the **fourth** derivative of $f(x) = \frac{1}{1-x}$ at $x = a$. Please label properly and simplify your final answer in terms of a . (2 pts.)

1.4 Find the absolute maximum and absolute minimum of the function $f(x) = \frac{2x-3}{x^2-2}$ on the closed interval $\left[0, \frac{5}{4}\right]$. (3 pts.)

SCORE

Problem 2: 10 points

2.1 Evaluate the definite integral $\int_1^4 5x\left(\sqrt{x} + \frac{1}{x^2}\right) + 5e^x dx$.

(3 pts.)

2.2 The marginal-cost function for a certain product is given by

$$\frac{dc}{dq} = 0.03q^2 - 0.8q + 3.4,$$

where c is a total cost expressed in dollars and q is the number of units produced. If the total cost of producing 10 units is \$346, find the total cost of producing 100 units. (3 pts.)

2.3 Let $f(x) = \frac{x^2 \ln(x^3 + 1)}{x^3 + 1}$. Find $\int_0^1 f(x) dx$. (2 pts.)

2.4 Let $f(x) = \frac{(\sqrt{x} - 1)^{10}}{\sqrt{x}}$. Find $\int_1^4 f(x) dx$. (2 pts.)

SCORE

Problem 3: 10 points

- 3.1 The supply and demand functions are given as $p = q^2 + 10$ and $p = 120 - q$, respectively. Draw a rough sketch of these two functions on the same qp -plane. Shade the region of the **producers' surplus** under the market equilibrium and find its value. (3 pts.)

- 3.2 Draw a rough sketch of the graphs $y = x^2$ and $y = 8 - x^2$ on the same xy -plane and find the area of the region bounded between these two curves. (3 pts.)

3.3 For a monopolist's product, the average cost function is

$$\bar{c} = 0.2q + 28 + \frac{200}{q}$$

and the demand function is

$$p = 600 - 2q.$$

Find the profit-maximizing output.

(4 pts.)

SCORE

Problem 4: 5 points

The following questions refer to the function $f(x) = (x^2 + 1)(x - 3)^2$.

4.1 Find all intercepts of the graph of this function. (1 pt.)

4.2 Determine the open intervals on which the function is increasing and the open intervals on which the function is decreasing. (2 pts.)

4.3 Determine the coordinates (x, y) of all relative extrema? Specify which point is a relative maximum and which is a relative minimum. (1 pt.)

4.4 Sketch the graph of this function including important points such as intercepts and relative extrema on the graph. (1 pt.)