

**Mahidol University International College****ICNS 103 Final Examination****12 April 2014 – 10.00 - 11.50****35 points**

Directions: Solve the following problems using the bottom of each page for scratch-work. Write up your solution and answer (in simplified form) in the space provided. Calculators are NOT allowed for this examination.

SCORE

Problem 1: 10 points

1.1 Consider the following function:

$$F(t) = \ln(2t + 1)^3 - \ln(3t - 1)^4.$$

(a) Find $F''(t)$.

(2 pts.)

(b) Find $F''(0)$.

(1 pt.)

1.2 Let $f(x, y, z) = x^2 e^{yz}$. Find:

(a) f_y (1 pt.)

(b) $f_y(1, 0, 1)$ (1 pt.)

1.3 A manufacturer's joint-cost function is given by $c = (2x + y)^{\frac{3}{2}}$. Find the marginal cost with respect to x when $x = 20$ and $y = 60$. (2 pts.)

1.4 Find the absolute maximum and absolute minimum of the function $f(x) = x^3 - 6x^2 + 9x + 2$ on the closed interval $[-1, 2]$. (3 pts.)

SCORE

Problem 2: 10 points

2.1 Evaluate $\int \left[\frac{2}{x^2} + \frac{5}{x} - 3e^2 + x^{4e} \right] dx.$

(2 pts.)

2.2 Evaluate $\int \left[5x^2 e^{x^3} - \frac{x \ln(x^2 + 10)}{x^2 + 10} \right] dx.$

(4 pts.)

2.3 Let $f'(x) = x\sqrt{x^2 + 1}$ and $f(0) = 4$. Find $f(x)$. (2 pts.)

2.4 Evaluate $\int_1^3 \left[\left(\frac{x^2 - 5}{\sqrt{x}} \right)^2 - \frac{25}{x} \right] dx$. (2 pts.)

SCORE

Problem 3: 10 points

3.1 The demand and supply equations are given, respectively, by

$$p = -\frac{q}{200} + 5 \quad \text{and} \quad p = \frac{q}{100} + \frac{1}{2}.$$

Draw a rough sketch of these two functions on the same qp -plane. Shade the region of **producers' surplus** under the market equilibrium and find its value. (3 pts.)

3.2 Draw a rough sketch of the graphs $y = 12 - x^2$ and $y = x^2 - 6$ on the same plane and find the area of the region bounded between these two curves. (3 pts.)

3.3 For a monopolist's product, the demand function is $p = \frac{50}{\sqrt{q}}$ and the average-cost function is $\bar{c} = \frac{1}{4} + \frac{2500}{q}$. Find the profit-maximizing price. (4 pts.)

SCORE

Problem 4: 5 points

The following questions refer to the function $f(x) = x^4 - 4x^3$.

4.1 Find all intercepts of the graph of this function. (1 pt.)

4.2 Determine where the function is increasing and where the function is decreasing. Determine the points (x, y) of all relative extrema? Specify which point is a relative maximum and which is a relative minimum. (2 pts.)

4.3 Determine where the function is concave up and where the function is concave down. Determine all inflection points. (1 pt.)

4.4 Sketch the graph of this function including important points such as intercepts, relative extrema, and inflection points on the graph. (1 pt.)