



**Mahidol University International College**

**ICNS 103**

**Fundamental Mathematics**

**Final Exam**

**Saturday, 19 July 2014**

**10.00 - 11.50**

**50 points**

**Directions:** Solve the following problems using the bottom of each page for scratch-work. Write up your solution and answer (in simplified form) in the space provided. A calculator is NOT allowed for this exam.

SCORE

**Problem 1: 10 points**

1.1 Determine all absolute extrema of  $p(x) = \frac{x^2 + 2}{2x - 1}$  on the interval  $[-2, 0]$ . (3 pts.)

1.2 Suppose  $g(t) = 4e^{1-t} - \frac{t^4}{4} - \ln(4t)$ .

Determine the **fourth derivative** of  $g(t)$  and evaluate it at  $t = 1$ . Label each line using proper notation and simplify your final answer to an integer. (4 pts.)

1.3 A function  $f(x)$  satisfies

$$f'(x) = e^x - 2x \quad \text{and} \quad f(1) = 2.$$

Determine  $f(x)$  and  $f(0)$ . (3 pts.)

SCORE

**Problem 2: 10 points**

2.1 Use the first or second derivative test to find all relative maxima and minima of the function

$$f(x) = \frac{x^3}{3} + 2x^2 - 5x + 1.$$

(3 pts.)

2.3 The following questions refer to the function

$$f(x) = 12x - x^3.$$

(a) Find all intercepts of the graph of this function.

(1 pt.)

(b) Determine intervals where the function is increasing and where the function is decreasing. Determine the points  $(x, y)$  of all relative extrema. Specify which point is a relative maximum and which is a relative minimum. (3 pts.)

(c) Determine intervals where the graph of function is concave up and where the graph is concave down. (2 pts.)

(d) Sketch the graph of this function including important points (intercepts and relative extrema) on the graph. (1 pt.)

SCORE

**Problem 3: 10 points**

3.1 Find the following indefinite integrals.

(a)  $\int \left( \sqrt{x} - \frac{1}{\sqrt{x}} \right) dx$  (2 pts.)

(b)  $\int \left( e^{2x} + \frac{1-x}{x^2} \right) dx$  (2 pts.)

(c)  $\int \frac{x-1}{2x^2-4x+5} dx$  (2 pts.)

3.2 If  $y$  is a function of  $x$  such that  $y' = \sqrt[3]{3x+1}$  and  $y(0) = 1$ , find  $y$ . (2 pts.)

3.3 Find the definite integral:  $\int_1^2 6 \left(x^2 - \frac{1}{x}\right) \left(x^3 - \frac{1}{x}\right) dx$  (2 pts.)

SCORE
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**Problem 4: 10 points**

4.1 For the given curves  $y = x^2$  and  $y = 2x$  :

(a) Sketch and shade the region bounded by the two curves and the lines  $x = 2$  and  $x = 3$ . (2 pts.)

(b) Find the area of the shaded region in part (a). (3 pts.)

- 4.2 The supply and demand functions are given by  $p = q^2 + 20$  and  $p = -q^2 + 220$ , respectively. Draw a rough sketch of these two functions, shade the region of the consumers' surplus, and evaluate its value. (5 pts.)



SCORE

**Problem 5: 10 points**

5.1 Find  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  where  $f(x, y) = 3x^2y - e^{5xy^2} + \ln(3x^2 + 4y^3)$ . (3 pts.)

5.2 The cost function for producing  $x$  units of product  $A$  and  $y$  units of product  $B$  is given by

$$c = \sqrt{x^2 + y^2} + 500y.$$

Find the marginal cost with respect to  $x$  when  $x = 30$  and  $y = 40$ . (3 pts.)

5.3 For a monopolist's product, the demand function is

$$p = 500 - \frac{100}{q}$$

and the total cost function is

$$c = q^2 + 300q.$$

Determine the level of output at which profit is maximized. Explain how you know that it will produce a maximum profit.

(Recall that **profit = total revenue - total cost.**) (4 pts.)