

**Mahidol University International College****ICNS 103****Fundamental Mathematics****Final Exam****Saturday, 6 December 2014****12.00 - 13.50****50 points**

Directions: Solve the following problems using the bottom of each page for scratch-work. Write up your solution and answer (in simplified form) in the space provided. A calculator is NOT allowed for this exam.

SCORE

Problem 1: 10 points

1.1 If $f(x) = x^2 \ln x$, find $f'(1) + f''(1) + f'''(1)$.

(2 pts.)

1.2 If $f(x, y) = \frac{4xy + xy^3}{x^2y + xy}$, find $f_x(1, 2)$.

(3 pts.)

- 1.3 A company's production function is given by $P = 15\ell^2k - 2\ell k^2 + 3k + 20$, where P is the total output generated by ℓ units of labor and k units of capital. Determine the marginal production function with respect to ℓ . (2 pts.)

- 1.4 A green tea company manufactures three flavored drinks (X , Y , and Z) and the joint-cost function for these products is given by $c = x^2\sqrt{y + 2z}$, where c is the total cost of producing x units of X , y units of Y , and z units of Z . Determine the marginal cost with respect to x when $x = 2, y = 1, z = 4$. (3 pts.)

SCORE

Problem 2: 10 points

2.1 The following questions refer to the function

$$f(x) = x^3 + 6x^2.$$

(a) Find all intercepts of the graph of this function. *(1 pt.)*

(b) Determine intervals where the function is increasing and where the function is decreasing. Determine the points (x, y) of all relative extrema. Specify which point is a relative maximum and which is a relative minimum. *(3 pts.)*

(c) Determine intervals where the graph of function is concave up and where the graph is concave down. *(2 pts.)*

- (d) Sketch the graph of this function including important points (intercepts, relative extrema, and inflection points) on the graph. (1 pt.)



2.2 Find all absolute extrema of the function

$$f(x) = \frac{x}{x^2 + 4}$$

on the closed interval $[0, 3]$.

(3 pts.)

SCORE

Problem 3: 10 points

3.1 For the cost function below:

$$c(q) = 25,000 + 120q + 0.1q^2$$

(a) Find the average cost at the production level of 1000 units. (1 pt.)

(b) Find the production level that will minimize the average cost. Your answer must be justified as why the level will minimize the average cost rather than maximize it. (2 pts.)

(c) Find the minimum average cost (1 pt.)

3.2 Evaluate the integral

$$\int \left(\frac{e^{4x} - e^x}{e^x} + \frac{7}{2x} \right) dx.$$

(3 pts.)

3.3 For the marginal-revenue function below:

$$\frac{dr}{dq} = 240 + 80q - 6q^2$$

(a) Find the demand function.

(2 pts.)

(b) What is the revenue when 10 units of the product are sold?

(1 pt.)

SCORE

Problem 4: 10 points

4.1 Find $\int \left(e^{3x+5} - \frac{(\ln(5x))^3}{x} \right) dx$

(3 pts.)

4.2 Find $\int x^3 \sqrt{x^2 + 1} dx$

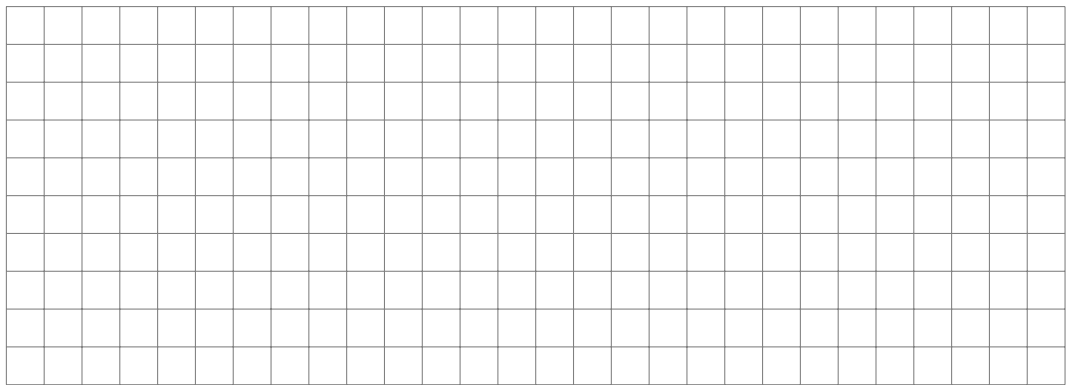
(3 pts.)

4.3 Find $\int_0^1 [4x(4x^2 + 9)^{10} + 2] dx$ (4 pts.)

SCORE

Problem 5: 10 points

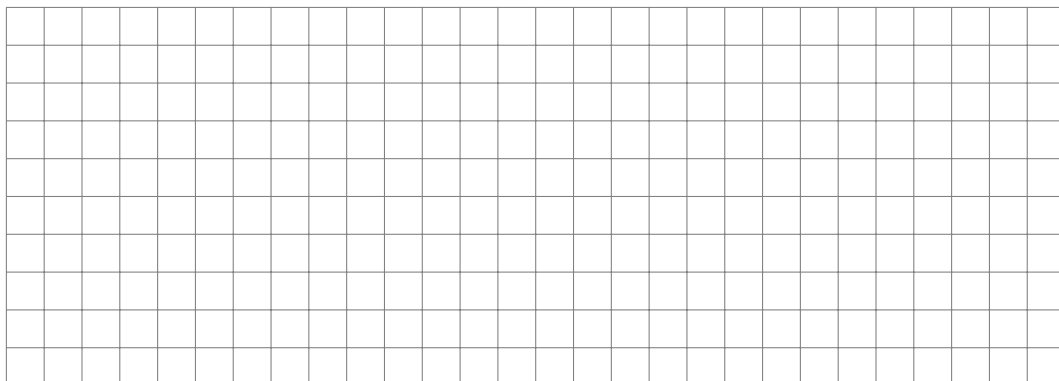
- 5.1 (a) Sketch the curves of $y = x^2 - 2x$ and $y = x$ on the same axes and shade the region bounded between these two curves. *(2 pts.)*



- (b) Find the area of the shaded region in part (a). *(2 pts.)*

5.2 The demand equation for a product is $p = (q - 5)^2$ and the supply equation is $p = (q + 1)^2$ where p (in hundreds of dollars) is the price per unit when q units are demanded or supplied.

- (a) Sketch the supply and demand curves on the same axes and determine the market equilibrium. *(2 pts.)*



- (b) Determine producers' surplus when market equilibrium has been established. *(4 pts.)*