



**Mahidol University International College**

**ICNS 103**

**Fundamental Mathematics**

**Final Exam**

**Saturday, 28 March 2015**

**12.00 - 13.50**

**80 points**

**Directions** Solve the following problems using the bottom of each page or any blank space for scratch-work. Answer the questions according to the instructions in each part. Write your name, ID number, section, and seat number in the space provided on each page. A calculator is NOT allowed for this exam.

SCORE

**Problem 1** 20 points

**Follow the instruction on the next page.**

Name----- ID #----- Section #----- Seat #-----

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SCORE

**Problem 2** 20 points

2.1 Find the integrals.

(a)  $\int \left( \sqrt[3]{x} + \frac{2}{\sqrt[3]{x}} - 5e^x + \frac{9}{x} - 10e \right) dx$  (5 pts.)

(b)  $\int \frac{\sqrt{x}(x+3)^2}{7x} dx$  (5 pts.)

2.2 Find absolute extrema for  $f(x) = 2x^3 - 15x^2 + 36x + 1$  over the closed interval  $[0, 3]$ . (5 pts.)

2.3 For a monopolist's product, the demand function is  $p = \frac{40}{\sqrt{q}}$  and the average cost function is  $\bar{c} = \frac{1}{3} + \frac{100}{q}$ . Determine the level of output at which profit is maximized. What is the maximum profit? (Use appropriate test of extrema to justify your answer.) (5 pts.)

SCORE

**Problem 3** 20 points

- 3.1 For a toy manufacturer, a marginal-revenue function is  $r' = 50 + 10q$  and a marginal-cost function is  $c' = 5q + 25$  with fixed costs of \$500. At production level of  $q = 10$ , will this manufacturer receive profit or face loss, if any? How much is the profit or loss? **Hint:** Profit = Revenue – Cost (5 pts.)

- 3.2 Find the indefinite integral:

(5 pts.)

$$\int \frac{x^2 + x - 12}{x - 3} dx.$$

3.3 Evaluate the definite integral:

(5 pts.)

$$\int_0^1 x^2(1 + 2x^3)^5 dx.$$

3.4 Find the indefinite integral:

(5 pts.)

$$\int x^5 \sqrt[3]{x^3 + 1} dx.$$



SCORE

**Problem 4** 20 points4.1 **Definite Integrals:** Let the function  $f$  be defined as follows

$$f(x) = \begin{cases} e^{-x}, & \text{if } x < 0, \\ 3x^2 + 1, & \text{if } x \geq 0. \end{cases}$$

Evaluate

*(2 pts. each)*

(a)

$$\int_{-1}^0 f(x) dx$$

(b)

$$\int_0^1 f(x) dx$$

(c)

$$\int_{-1}^1 f(x) dx$$

4.2 **Area between Curves:** Let the equations of the two curves be given by

$$y_1 = 1 - x^2 \quad \text{and} \quad y_2 = x + 1.$$

(a) Determine the coordinates where the two curves intersect.

*(2 pts.)*

(b) Write the expression to determine the area bounded by the two curves. (3 pts.)

(c) Calculate the expression obtained in part (b). (2 pts.)

**4.3 Consumers' and Producers' Surplus:** The demand equation and the supply equation for a product are given, respectively, by

$$p = 5 - \frac{q}{200} \quad \text{and} \quad p = \frac{q}{100} + \frac{1}{2}.$$

(a) Determine the price and quantity (point of equilibrium) at market equilibrium. (2 pts.)

(b) Write the mathematical expression to determine the consumers' surplus. (3 pts.)

(c) Evaluate the expression obtained in part (b). (2 pts.)