



Mahidol University International College

ICNS 103

Fundamental Mathematics

Final Exam

Saturday, 18 July 2015

10.00 - 11.50

45 points

Directions Solve the following problems using the bottom of each page or any blank space for scratch-work. Answer the questions according to the instructions in each part. Write your name, ID number, section, and seat number in the space provided on each page. A calculator is NOT allowed for this exam.

SCORE

Problem 1 5 points

1.1 Let $\lim_{x \rightarrow 0} f(x) = 5$. Find $\lim_{x \rightarrow 0} \frac{xf(x) - x}{x}$.

(1 pt.)

1.2 Let $f(x) = xe^{x^2-x}$. Find the rate of change of $f(x)$ when $x = 1$.

(1 pt.)

1.3 Find an equation of the tangent line to the curve $xy = x + 2y^2$ at $(-1, -1)$. (2 pts.)

1.4 Let $f(x) = \frac{1}{2x - 1}$. Find $f''(1)$. (1 pt.)

SCORE

Problem 2 10 points

2.1 A company manufactures two products, X and Y , and the joint-cost function for these products is given by $c = 7x + 3y^2 + 2xy + 800$, where c is the total cost of producing x units of X and y units of Y . Determine the marginal cost with respect to y when $x = 20$ and $y = 30$. (2 pts.)

2.2 Use the second derivative test to find relative maxima and minima of the function

$$f(x) = -x^3 + 3x^2 + 1.$$

(2 pts.)

2.3 Given $f(x) = x^2 - 3x - 10$,

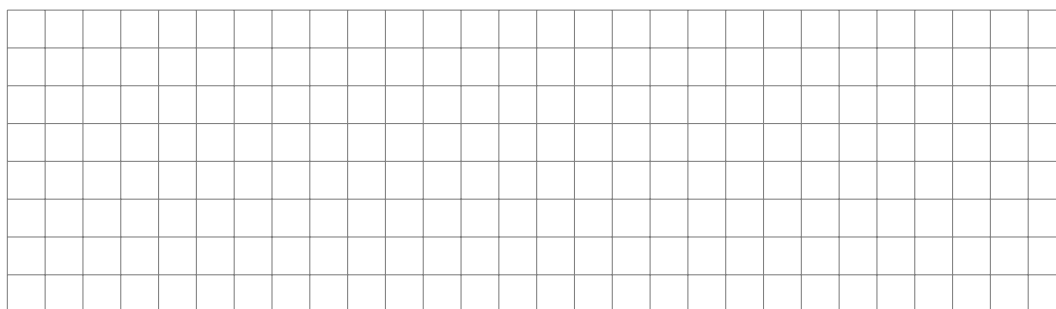
(a) determine where the function is increasing or decreasing, (1 pt.)

(b) determine where the relative maxima and minima occur, (1 pt.)

(c) determine where the function is concave up or concave down, (1 pt.)

(d) determine the function intercepts. (1 pt.)

2.4 Sketch one possible graph of a continuous function f based on the following information:
 $f(2) = 3$, $f'(2) = 0$, $f''(x) < 0$ for all x in $(-\infty, 2)$ and $f''(x) > 0$ for all x in $(2, \infty)$. (2 pts.)



SCORE

Problem 3 10 points

3.1 Find the absolute extrema for $f(x)$ over the closed interval $[-2, 0]$: (3 pts.)

$$f(x) = 2x^3 + 2x^2 - 2x - 3.$$

3.2 Find the indefinite integrals:

(a) $\int \frac{x^3 - 9x}{x - 3} dx.$ (2 pts.)

(b) $\int (2\sqrt[3]{x^2} - 3x^2 + 2e^x + 5) dx.$ (2 pts.)

3.3 The demand function for a monopolist's product is $p = 900 - 5q$ and the average cost per unit for producing q units is $\bar{c} = q + 60 + \frac{9400}{q}$ where p and \bar{c} are in dollars per unit. Find the maximum profit that the monopolist can achieve. (3 pts.)

SCORE

Problem 4 10 points

4.1 Suppose the marginal cost function for a product is

$$\frac{dc}{dq} = -30(q + 1)^2 + 300e^{q/3}$$

with fixed costs $c(0) = 2000$ baht. Find the total cost function and then use it to determine whether a profit is made at $q = 3$ if total revenue is 4000 baht. Explain. You may use $e \approx 3$ to approximate your answers. (4 pts.)

4.2 Determine the following indefinite integrals:

(a) $\int \frac{\ln(x) - 3}{x} dx$ (3 pts.)

(b) $\int \frac{t}{\sqrt{6 - 3t^2}} dt$ (3 pts.)

SCORE

Problem 5 10 points

5.1 Evaluate the definite integral:

$$\int_0^1 3y\sqrt{4-3y^2} dy.$$

(3 pts.)

5.2 The demand equation for a product is $p = (q - 10)^2$. Find consumers's surplus under market equilibrium, which occurs at a price of \$25. (3 pts.)

5.3 Sketch the region bounded by the curves $y = x^2 - 2x$ and $y = 3$ and find its area. (4 pts.)

