



**Mahidol University International College**  
**Trimester 1 Academic Year 2015-16**

|                                   |                                |                   |
|-----------------------------------|--------------------------------|-------------------|
| <b>ICNS 103</b>                   | <b>Fundamental Mathematics</b> | <b>Final Exam</b> |
| <b>Saturday, 12 December 2015</b> | <b>12.00 - 13.50</b>           | <b>45 points</b>  |

**Directions** Solve the following problems using the bottom of each page or any blank space for scratch-work. Answer the questions according to the instructions in each part. Write your name, ID number, section, and seat number in the space provided on each page. A calculator is NOT allowed for this exam.

**Problem 1 5 points**

|              |
|--------------|
| <b>SCORE</b> |
|              |

1.1 Find  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + x - 2}$ .

*(1 pt.)*

1.2 Let  $f(x) = (x - \sqrt{x})^4$ . Find the rate of change of  $f(x)$  when  $x = 4$ .

*(1 pt.)*

1.3 Find an equation of the tangent line to the curve  $6x^2 + y^2 = 10$  at  $(1, 2)$ . *(1 pt.)*

1.4 Let  $f(x) = \sqrt{x^2 + 1}$ . Find  $f''(x)$  in the form  $\frac{a}{(x^2 + 1)^{\frac{3}{2}}}$  for some constant  $a$ . *(2 pts.)*

|              |
|--------------|
| <b>SCORE</b> |
|              |

**Problem 2** 10 points

2.1 Fill in the blank with appropriate answer. Let  $y = x^3 + 15x^2 - 33x$ .

(2 pts.)

- (a) The function has a relative maximum when  $x = \dots\dots\dots$
- (b) The function has a relative minimum when  $x = \dots\dots\dots$
- (c) The curve of the function is concave  $\dots\dots\dots$  on  $(-\infty, -5)$ .
- (d) The curve of the function is concave  $\dots\dots\dots$  on  $(-5, \infty)$ .

2.2 Find all critical values of the following function:

$$f(x) = \frac{x + 2}{x^2 - 3}$$

(2 pts.)

2.3 The production function of a manufacturer is given by

$$P = 1000 + 20\ell^2k^3 - 5\ell^3 - 3k^4,$$

where  $\ell$  is labor in thousand labor-hours per week,  $k$  is capital in thousand dollars per week, and  $P$  is production in thousand grosses per week. *(3 pts.)*

(a) Find all marginal productivity functions.

(b) What is the marginal productivity with respect to  $\ell$  when  $\ell = 1$  and  $k = 2$ ? Make sure to include the units in your final answer.

2.4 Find the  $x$ -coordinates of all inflection points of the graph of  $y = x(x - 1)^3$ . *(3 pts.)*

|              |
|--------------|
| <b>SCORE</b> |
|              |

**Problem 3** 10 points

3.1 Find the absolute extrema for  $f(x)$  over the closed interval  $[-10, 0]$  :

(3 pts.)

$$f(x) = x\sqrt{x + 12}.$$

(Recall that  $\sqrt{2} \approx 1.4$ .)

3.2 Find the indefinite integral:  $\int \frac{(x + 1)^2}{\sqrt{x}} dx$ .

(2 pts.)

3.3 If  $\int f(x) dx = x^3 - 3x^2 + C$ , what is the value of  $f(2)$ ? (2 pts.)

3.4 Suppose a particular monopolist estimates that when  $q$  units are produced, the total cost will be

$$c = \frac{7}{8}q^2 + 5q + 100$$

dollars and the market price of the commodity will be

$$p = 15 - \frac{3}{8}q$$

dollars per unit. Find the price that will maximize the profit. (3 pts.)

| SCORE |
|-------|
|       |

**Problem 4** 10 points

4.1 Find the function  $y$  satisfying  $y'' = e^{x-5} + 1$ ,  $y'(5) = 7$ , and  $y(5) = 6$ .

(4 pts.)

4.2 Determine the following indefinite integrals:

(a)  $\int \frac{(\sqrt{x} + 2)^5}{3\sqrt{x}} dx$  (3 pts.)

(b)  $\int \frac{5x^3}{x^2 + 9} dx$  (3 pts.)



|              |
|--------------|
| <b>SCORE</b> |
|              |

**Problem 5** 10 points

5.1 Evaluate the definite integral:

$$\int_0^2 \frac{dx}{(2x+3)^2}$$

*(3 pts.)*

5.2 The demand equation for a product is  $p = 50 - \frac{q}{20}$  and the supply function is  $p = 20 + \frac{q}{10}$ , where  $p$  is in dollars. Find consumers' surplus under the market equilibrium. *(3 pts.)*

5.3 Sketch and shade the region bounded by the curves  $y = 12 - x^2$ ,  $y = x^2 - 6$ ,  $x = -3$ , and  $x = 2$  and find its area. (4 pts.)

