



Mahidol University International College  
Trimester 2 Academic Year 2015-16

ICNS 103	Fundamental Mathematics	Final Exam
Saturday, 2 April 2016	8.00 - 9.50	45 points

**Directions** Solve the following problems using the bottom of each page or any blank space for scratch-work. Answer the questions according to the instructions in each part. Write your name, ID number, section, and seat number in the space provided on each page. A calculator is NOT allowed for this exam.

**Problem 1** 5 points

SCORE
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1.1 Find  $\lim_{x \rightarrow 2} \frac{1}{x-2} \left( \frac{x}{2} - \frac{2}{x} \right)$ .

(1 pt.)

1.2 If it is known that the two functions  $f(x) = 3x^2 + ax$  and  $g(x) = 9 \ln x - 2ax$  have the same slope at  $x = 3$ , then what is the value of  $a$ ?

(1 pt.)

1.3 Find an equation of the tangent line to the curve  $\frac{1}{x} + \frac{4}{y^2} = xy$  at the point  $(1, 2)$ . (1 pt.)

1.4 Let  $f(x) = (x + 1)\sqrt{x}$ . If  $f''(a) = 0$ , then what is the value of  $a$ ? (2 pts.)

SCORE

**Problem 2** 10 points

2.1 Find  $\frac{\partial f}{\partial x}$  of the following function:  $f(x, y) = e^{xy} \ln y$ .

(2 pts.)

2.2 Assuming Honda USA manufactures two car models, Civic and HRV. Suppose the joint-cost function for producing  $x$  cars of the Civic model and  $y$  cars of the HRV model is:

$$c = f(x, y) = (x + 1)^2 + (y - 3)^3 + 5xy^3 - 2,$$

where  $c$  is expressed in thousand dollars.

(a) Determine the marginal costs  $\frac{\partial c}{\partial x}$  and  $\frac{\partial c}{\partial y}$  when  $x = 10$  and  $y = 10$ .

(2 pts.)

(b) Based on part (a), if both models can be sold for \$20,000 and the manufacturer can produce only one more car from its plant, what model should the manufacturer choose to produce and sell? Justify your answer.

(1 pt.)

2.3 Given  $f(x) = x^4 + 4x^3$ ,

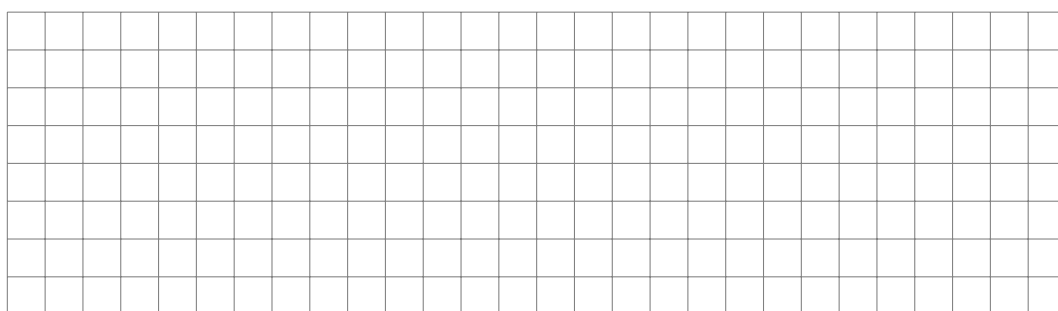
(a) Find  $x$  and  $y$ -intercepts. (1 pt.)

(b) Determine the points of all relative extrema. Specify which point is a relative maximum and which is a relative minimum. (1 pt.)

(c) Determine where the function is increasing or decreasing. (1 pt.)

(d) Determine where the function is concave up or concave down. (1 pt.)

(e) Sketch the graph based on the information obtained above. (1 pt.)



SCORE

**Problem 3** 10 points

3.1 Find all absolute extrema for  $f(x)$  over the closed interval  $[-1, 2]$  :

(3 pts.)

$$f(x) = \frac{1}{x^2 + 1}.$$

3.2 Find the indefinite integral:  $\int \left( 2e^x - 3x^5 + 4e^5 - \frac{5}{x} \right) dx.$

(1 pt.)

3.3 Find the indefinite integral:  $\int \frac{(2x-1)^2}{3\sqrt{x}} dx$ . (2 pts.)

3.4 The demand function  $p$  for a monopolist's product is

$$p = \frac{10}{\sqrt{q}}$$

and the average cost per unit is

$$\bar{c} = \frac{1}{3} + \frac{20}{q},$$

where  $\bar{c}$  and  $p$  are in dollars per unit. Find the maximum profit that the monopolist can achieve. (4 pts.)

<b>SCORE</b>

**Problem 4** 10 points

4.1 The marginal revenue function is given by

$$\frac{dr}{dq} = 3q^2 - 60q + 225.$$

We assume that when no units are sold, there is no revenue, that is  $r(0) = 0$ .

(a) Determine the total revenue function. *(2 pts.)*

(b) Find the price per unit for selling 10 units of output. *(2 pts.)*

4.2 Find the function  $y$  satisfying

$$y'(x) = \frac{2x \ln(1 + x^2)}{1 + x^2},$$

and  $y(0) = 2$ .

(3 pts.)

4.3 Determine the following indefinite integral

$$\int x^7(x^4 + 3)^{10} dx.$$

(3 pts.)



SCORE

**Problem 5** 10 points

5.1 Evaluate the definite integral:

$$\int_0^8 \frac{2}{\sqrt{9-x}} dx.$$

*(3 pts.)*

5.2 The demand equation for a product is  $p = 4 - q$  and the supply function is  $p = \frac{1}{2}q^2$ , where  $p$  is in thousands of dollars per unit. Find producers' surplus under the market equilibrium. *(3 pts.)*

5.3 Find the area bounded by  $y = 4 - x^2$  and  $y = x^2 - 4$  on the interval  $[0, 3]$ . Also, sketch the two curves and shade the area between them over this interval. (4 pts.)

