



Mahidol University International College
Trimester 3 Academic Year 2015-16

ICNS 103	Fundamental Mathematics	Final Exam
Saturday, 23 July 2016	12.00 - 13.50	45 points

Directions Solve the following problems using the bottom of each page or any blank space for scratch-work. Answer the questions according to the instructions in each part. Write your name, ID number, section, and seat number in the space provided on each page. A calculator is NOT allowed for this exam.

SCORE

Problem 1 5 points

1.1 Find $\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{x^2 + 1}{2} - \frac{x + 3}{6} \right)$.

(1 pt.)

1.2 Functions $f(x)$ and $g(x)$ are related by

$$f(x) = \frac{x}{x^2 + g(x)}.$$

If $g(2) = g'(2) = -3$, then what is the value of $f'(2)$?

(1 pt.)

1.3 Find the slope of the tangent line to the curve $-2y^2x^3 = 9(x - y)$ at the point $(1, 3)$. *(1 pt.)*

1.4 Let $f(x) = e^{2x} + \ln(x^2 + 1)$. Find $f''(x)$ in simplest form. *(2 pts.)*

SCORE

Problem 2 10 points

2.1 Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of the following function: $f(x, y) = 2\sqrt{x^5 + y^2 - x}$.

(2 pts.)

2.2 A company manufactures two types of hand-made watches, the Prestige model and the Elegant model. Suppose the joint-cost function for producing x watches of the Prestige model per week and y watches of the Elegant model per week is:

$$C = f(x, y) = x^4y^3 + 8x^2y,$$

where C is expressed in **baht**. Suppose also that the company can sell all the watches it produces in a week.

(a) Determine the marginal costs $\frac{\partial C}{\partial x}$ and $\frac{\partial C}{\partial y}$ when $x = 10$ and $y = 10$, and interpret the results. (2 pts.)

(b) Assume that both watch models have the same price tag of 4,500,000 baht and the company typically produces 10 watches of each model each week. If the company decides to produce one more watch this week, which model should it choose to produce? Why? (1 pt.)

2.3 Given $f(x) = (x^2 + 1)(x - 2)$,

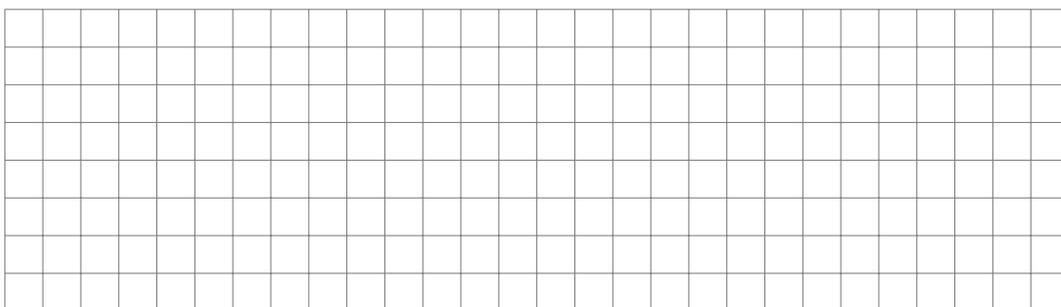
(a) Find x and y -intercepts. (1 pt.)

(b) Determine the points of all relative extrema. Specify which point is a relative maximum and which is a relative minimum. (1 pt.)

(c) Determine where the function is increasing or decreasing. (1 pt.)

(d) Determine where the function is concave up or concave down. (1 pt.)

(e) Sketch the graph based on the information obtained above. (1 pt.)



SCORE

Problem 3 10 points

3.1 Let $f(x) = x^2\sqrt{x-5}$.

(a) Find all critical values of $f(x)$.

(2 pts.)

(b) Find all absolute extrema of $f(x)$ over the interval $[5, 6]$.

(2 pt.)

3.2 Find the indefinite integral: $\int (\sqrt{x} - 2)^2 dx$.

(1 pt.)

3.3 Find the indefinite integral: $\int \frac{\sqrt{2x} - \sqrt{3x^3}}{x^3} dx.$ (2 pts.)

3.4 At a scout camp, a student is assigned to use a 3,600-ft rope to create a rectangular area in a field next to a stream, using the stream as one of its sides. Therefore, there is no need to use the rope for the stream side of the area. What are the dimensions of the largest possible area that the student can create? (3 pts.)

SCORE

Problem 4 10 points4.1 Let f be a continuous function on the interval $[1, 7]$. Given that

$$\int_1^5 f(x) dx = 7, \quad \int_3^5 f(x) dx = 3, \quad \int_5^7 f(x) dx = 4,$$

find each of the following:

(a) $\int_5^5 f(x) dx$ (0.5 pt.)

(b) $\int_5^3 f(x) dx$ (0.5 pt.)

(c) $\int_1^7 f(x) dx$ (1 pt.)

(d) $\int_1^5 \left(\int_5^7 f(t) dt \right) dx$ (1 pt.)

4.2 Find the function f satisfying

$$f'(x) = 2e^{2x+1} - 3e^{3x+1},$$

and $f(0) + f(1) = 4 + e^3 - e^4$.

(3 pts.)

4.3 Determine the following indefinite integral

$$\int x^3 \sqrt{x^2 + 5} dx.$$

(4 pts.)

SCORE

Problem 5 10 points

5.1 Evaluate the definite integral:

$$\int_0^1 (x^2 - 1)(x^3 - 3x)^4 dx.$$

(3 pts.)

5.2 The demand equation for a monopolist's product is $p = (q - 4)^2$ and the supply equation is $p = (2q + 1)^2$, where p is in **thousands** of dollars per unit. Find the consumers' surplus under the market equilibrium.

(3 pts.)

5.3 Sketch the two curves $y = 2x - x^2$ and $y = -3$, shade the region bounded by these two curves, and find the area of this region. (4 pts.)

