Mahidol University International College

ICNS 103 Midterm Examination

1 March 2014 – 10.00 - 11.50   **35 points**

Directions: Solve the following problems using the bottom of each page for scratch-work. Write up your solution and answer (in simplified form) in the space provided. Calculators are NOT allowed for this examination.

SCORE

Problem 1: 10 points

1.1 Find the following limits:

(a) $\lim\limits_{x \to 3} \dfrac{2x^2 - 5x - 3}{x^2 + 2x - 15} =$ ....   (1 pt.)

(b) $\lim\limits_{x \to \infty} \dfrac{(4x - 3)^2}{7x^2 + x + 5} =$ ....   (1 pt.)

1.2 Let $f(x) = \begin{cases} kx^2 + 5x - 2, & \text{if } x < -2; \\ 3kx - 2, & \text{if } x \geq -2. \end{cases}$ Answer the following questions.

(a) $\displaystyle\lim_{x \to -2^+} f(x) = \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$ (1 pt.)

(b) $\displaystyle\lim_{x \to -2^-} f(x) = \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$ (1 pt.)

(c) Find the value of k so that $\displaystyle\lim_{x \to -2} f(x)$ exists.   (1 pt.)

(d) With the value of k obtained from part (c), find $f(-2)$.   (1 pt.)

1.3 Use the definition of continuity to determine whether the following function is continuous at 1.

$$f(x) = \begin{cases} 4x^2 - 5x + 6, & \text{if } x > 1; \\ 3, & \text{if } x = 1; \\ \dfrac{3x + 7}{x^2 + 1}, & \text{if } x < 1. \end{cases}$$

(4 pts.)

       scratch-work

SCORE

Problem 2: 10 points

2.1 Let $f(x) = -\dfrac{1}{x+1}$. Use the definition of the derivative to find $f'(x)$. (2 pts.)

2.2 Let $g(x) = \dfrac{3x^2 + 4x - 2}{x} + 7^5$. Evaluate $g'(2)$.

(2 pts.)

scratch-work

2.3 Find all x-coordinates on the curve $y = \dfrac{x^6}{6} - 8x^2 + 1$ where the tangent line is horizontal.

(3 pts.)

2.4 Find an equation of the tangent line to the curve $y = 3x - 4\sqrt{x}$ at $x = 4$. (3 pts.)

scratch-work

SCORE

Problem 3: 10 points

3.1 Differentiate the following functions with respect to x. Simplify your result.

(a) $f(x) = \sqrt{(2-x)^2 + 3(2-x)^2}$ for $x < 2$ (2 pts.)

(b) $g(x) = \dfrac{2}{3\sqrt{x^3 - 1}} - \dfrac{3x(4 + \sqrt{x})^4}{2}$ (3 pts.)

3.2 Suppose that

$$y = \sqrt[3]{u}, \quad u = 16 - \frac{1}{t^3}, \quad \text{and} \quad t = 1 - x.$$

Determine $\left. \dfrac{dy}{dx} \right|_{x=1/2}$. Simplify your answer to an integer. (2 pts.)

3.3 Suppose that $r(t) = \dfrac{(t-1)^9}{5(t+2)}$ gives the total revenue (in baht) after t days of sales. Find the rate of change of the revenue with respect to time after the **second** day. Include proper units in your final answer. (3 pts.)

scratch-work

SCORE

Problem 4: 5 points

4.1 Let $y = \dfrac{e^x}{(1 + e^x)^2}$. Find $\dfrac{dy}{dx}$. Simplify your result. (2 pts.)

4.2 Let $y = x(\ln x)^2 + x \ln x$. Find $\dfrac{dy}{dx}\Big|_{x=e}$. (2 pts.)

4.3 The cost function is given by $c(q) = e^{3q-2}$, where c is in dollars. Find the marginal cost when $q = 2$ and interpret the result. (Use $e \approx 3$ to approximate your answer.) (1 pt.)

scratch-work