



Mahidol University International College

ICNS 103

Fundamental Mathematics

Midterm Exam

Saturday, 7 June 2014

10.00 - 11.50

40 points

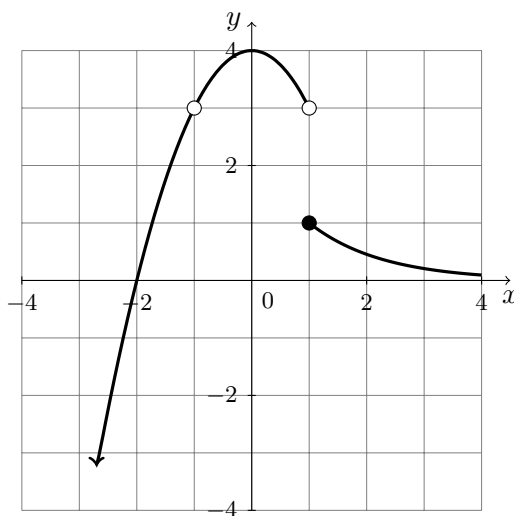
Directions: Solve the following problems using the bottom of each page for scratch-work. Write up your solution and answer (in simplified form) in the space provided. Calculator is NOT allowed for this exam.

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Problem 1 : 10 points

1.1 The graph of a function $f(x)$ is given below.

(6 pts.)



Find the following:

- (a) $\lim_{x \rightarrow 1^+} f(x)$
- (b) $\lim_{x \rightarrow 1^-} f(x)$
- (c) $\lim_{x \rightarrow 1} f(x)$
- (d) $\lim_{x \rightarrow \infty} f(x)$
- (e) $\lim_{x \rightarrow -\infty} f(x)$
- (f) all points of discontinuity

1.2 Let $g(x) = \frac{(3x - 2)^2}{3x^2 + x - 2}$.

(a) Find all points of discontinuity of the function g . (1 pt.)

(b) Find $\lim_{x \rightarrow -\infty} g(x)$. (1 pt.)

1.3 Use the definition of continuity to determine whether the following function is continuous at 2.

$$f(x) = \begin{cases} \frac{1}{2}x^3, & \text{if } x < 2; \\ 1, & \text{if } x = 2; \\ 2x, & \text{if } x > 2. \end{cases}$$

(2 pts.)

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Problem 2: 10 points

2.1 Let $f(x) = 2x^2 - 3x$. Use the definition of the derivative to find $f'(x)$. (2 pts.)

2.2 Let $g(x) = \frac{4x^2 - 2}{\sqrt{x}} - \frac{x}{8} + e^5$. Evaluate $g'(4)$ **without** using the quotient rule or the product rule of differentiation.

(2 pts.)

2.3 Find all points on the graph of $y = \frac{x^3}{3} - 2x^2 + 5x$ whose slope is 2. (3 pts.)

2.4 Find the derivative of the function $y = \frac{\sqrt{x}(16 - x^2)}{x}$ at $x = 4$ **without** using the product rule or the quotient rule of differentiation. (3 pts.)

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Problem 3: 10 points3.1 Answer the following questions. **Simplify** your results.

(a) Let $f(x) = \sqrt[3]{8(2x^3 - 1)^2}$. Find $f'(x)$. (2 pts.)

(b) Let $g(x) = \frac{x}{\sqrt{2x - 1}}$. Find $g'(x)$ and the slope of the function at $x = 5$. (3 pts.)

(c) Let $h(t) = (t^3 - 1)(t^2 + 1) + (t^3 + 1)(t^2 - 1)$. Find the rate of change of h . (2 pts.)

3.2 The average cost function \bar{c} in dollars/unit of a manufacturer is given by

$$\bar{c} = 0.002q^3 - 0.02q + \frac{10}{q}.$$

(a) Find the total cost function c . (1 pt.)

(b) Find the marginal cost when $q = 10$ and **interpret** the result. (2 pts.)

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Problem 4: 10 points

4.1 Let $y = 3u^2 - 8u + 4$ and $u = \frac{x+1}{x-1}$. Find $\frac{dy}{dx}$ when $x = 2$. (4 pts.)

4.2 Find an equation of the tangent line to the curve $y = 2x + \ln(x^2 - 3x - 3)$ at $x = 4$. (4 pts.)

4.3 Find $\frac{dy}{dx}$ if $y = (e^{5x+1} + e^{x^2+6x+7})^4$. (2 pts.)