Name _____ ID # _____ Section # _____ Seat # _____

Mahidol University International College

ICNS 103 Fundamental Mathematics Midterm Exam

Saturday, 25 October 2014 14.00 - 15.50 40 points

Directions: Solve the following problems using the bottom of each page for scratch-work. Write up your solution and answer (in simplified form) in the space provided. A calculator is NOT allowed for this exam.

SCORE

Problem 1: 10 points

1.1 Find the limits.

(a) $\lim_{x \to -2} \dfrac{x^3 + 8}{x + 2}$ *(2 pts.)*

(Hint: $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$)

(b) $\lim_{x \to \infty} \dfrac{(4x - 3)^2}{7x^2 + x + 5}$ *(2 pts.)*

1.2 Let $f(x) = \begin{cases} kx^2 - 5x + 1, & \text{if } x < -1, \\ 3kx + 8, & \text{if } x \geq -1. \end{cases}$

(a) Find $\lim_{x \to -1^+} f(x)$. *(1 pt.)*

(b) Find $\lim_{x \to -1^-} f(x)$. *(1 pt.)*

(c) Find the value of k so that $\lim_{x \to -1} f(x)$ exists. *(1 pt.)*

1.3 Use the definition of continuity to justify whether the function is continuous at 1.

$$f(x) = \begin{cases} \dfrac{5x - 1}{x^2 + 1}, & \text{if } x \geq 1, \\ \dfrac{x^2 - 1}{x - 1}, & \text{if } x < 1 \end{cases}$$

(3 pts.)

SCORE

Problem 2: 10 points

2.1 How many of the following statements are true? Put your answer in the dotted line below.

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(1 pt.)

- There is no tangent lines to the graph of $y = |x|$ at $(0, 0)$.
- If f is differentiable at a, then f is continuous at a.
- $y = \sqrt{x}$ is a function with a vertical tangent line at $(0, 0)$.
- A secant line is a line that intersects a curve at two or more points.
- The slope of a curve at a point P is the slope, if it exists, of the tangent line at P.

No partial credit is given for this question.

2.2 Use the definition of the derivative to find $f'(x)$ if $f(x) = x + e^{\pi}$.
(Hint: $e = 2.7...$, $\pi = 3.1...$) *(3 pts.)*

2.3 Differentiate the function $\psi = 100x^{-3} + 10x^{1/2}$ with respect to x. *(3 pts.)*

2.4 Find all points on the curve $y = (x - 5)^2$ where the slope is 10. *(3 pts.)*

SCORE

Problem 3: 10 points

3.1 For a certain book publisher, the average cost equation is given by $\bar{c} = 500 + 15q + 0.3q^2$, where the unit is in baht and q denotes the number of books printed. Answer the following questions.

(a) What is the total cost of printing 100 books. *(1 pt.)*

(b) What is the average cost per book when 100 books are printed? *(1 pt.)*

(c) What is the marginal cost when 100 books are printed? *(1 pt.)*

(d) Interpret the answer in (c). *(1 pt.)*

(e) What is the exact cost of printing the 101st book? *(1 pt.)*

3.2 Given $f(x) = (2x^2 + 4\sqrt{x})(8\sqrt{x} - 7)$, find $f'(1)$ by using the product rule. *(2 pts.)*

3.3 Let $h(x) = \dfrac{3x^2 + 2}{2x - x^2}$. Find an equation of the tangent line to the curve of $h(x)$ when $x = 1$. *(3 pts.)*

SCORE

Problem 4: 10 points

4.1 Let $y = u^2 - u + 1$ and $u = e^x - e^{-x}$. Find $\dfrac{dy}{dx}$ when $x = 0$. *(3 pts.)*

4.2 Let $f(x) = \ln\left(\dfrac{x^2 - 1}{x}\right)^3$. Find $f'(3)$. *(2 pts.)*

4.3 Let $y = e^{2x}(5 - x) - 2^{5-2\sqrt{x}}$. Find $\dfrac{dy}{dx}$ at $x = 4$. *(3 pts.)*

4.4 The tangent line to the curve $y = x\,(\ln x + k)$ at $x = e$ is parallel to the line $y = x$. Find the value of k. (Hint: $\ln e = 1$.) *(2 pts.)*