



Mahidol University International College

ICNS 103

Fundamental Mathematics

Midterm Exam

Saturday, 14 February 2015

10.00 - 11.50

80 points

Directions Solve the following problems using the bottom of each page or any blank space for scratch-work. Answer the questions according to the instructions in each part. Write your name, ID number, section, and seat number in the space provided on each page. A calculator is NOT allowed for this exam.

SCORE

Problem 1 20 points

Part I: Multiple Choice Questions (2 points each)

Circle the correct answer. Justification is not required.

1.1 Provide an appropriate response to the following limit: $\lim_{x \rightarrow -2^+} \frac{\pi}{x + 2}$.

- (a) π (b) 0 (c) $+\infty$ (d) $-\infty$ (e) $\frac{\pi}{2}$

1.2 Provide an appropriate response to the following limit: $\lim_{t \rightarrow 3} \frac{t^2 - t - 6}{3t^2 - 8t - 3}$.

- (a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $+\infty$ (e) Does not exist

1.3 Provide an appropriate response to the following limit: $\lim_{x \rightarrow 378.52} \frac{(x + 2)^2}{2x^2 + 8x + 8}$.

- (a) 0 (b) $\frac{1}{2}$ (c) 9263.48 (d) $+\infty$ (e) Does not exist

1.4 Provide an appropriate response to the following limit: $\lim_{x \rightarrow -\infty} \frac{x + 2}{2x^2 - 2x + 1}$.

- (a) 0 (b) $\frac{1}{2}$ (c) $+\infty$ (d) $-\infty$ (e) Does not exist

Part II: Open Response Questions (6 points each)

Write up the solution of each problem.

1.5 Find all points of discontinuity for the following function. Justify your answer.

$$f(x) = \begin{cases} \frac{2x}{x^2 - 5x + 6}, & \text{if } x \geq 1, \\ -5x, & \text{if } x < 1 \end{cases}$$

1.6 Let $a = \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$ and $f(x) = \begin{cases} \frac{x^8 - 2x^7}{x^6 - 2x^5}, & \text{if } x > a, \\ 3x^2 - 8, & \text{if } x \leq a. \end{cases}$

Find $\lim_{x \rightarrow a} f(x)$.

SCORE

Problem 2 20 points

2.1 Let $f(x) = 2x^2 + e^3 - 5x$, find $f'(x)$ by using the definition of the derivative. (This is the only question you need to use the definition to find the derivative.) (5 pts.)

2.2 Let $f(x) = 9x^3 - \sqrt{x} + \frac{4}{x^2} - 3\pi^2$. Find $f'(2)$. (5 pts.)

2.3 Let $f(x) = 2x^2 - 5x + 10$. Find the following:

(a) Average rate of change of f with respect to x from $x = 1$ to $x = 3$ (2 pts.)

(b) Rate of change of f with respect to x at $x = 1$ (1 pt.)

(c) The value of x such that the slope of f is zero (2 pts.)

2.4 (a) Let c be a total cost function of producing q units of output. What is the definition of the marginal cost and how do we interpret it? (2 pts.)

(b) A manufacturer of computers has found that when 10 computers are produced per day, the average cost is \$750 and the marginal cost is \$620. Based on that information, approximate the total cost of producing 11 computers per day. (3 pts.)

SCORE

Problem 3 20 points

3.1 Fill in an appropriate response for each of the following: (4 pts.)

(a) $\frac{d}{dx}(f(x)g(x)) = \dots\dots\dots$ (b) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \dots\dots\dots$

(c) $\frac{d}{dx}e^{x^2} = \dots\dots\dots$ (d) $\frac{d}{dx}\ln(2x + 1) = \dots\dots\dots$

3.2 Let $y = 2e^u + e^{-u}$ and $u = x \ln x$. Find $\frac{dy}{dx}$ when $x = 1$. (4 pts.)

3.3 Given $f(x) = (4\sqrt{x} - 3)(2\sqrt{x} + x)$, find $f'(4)$ by using the product rule. (4 pts.)

3.4 Let $h(x) = \frac{x^2 + 3}{x^2 + 1}$. Find an equation of the tangent line to the curve of $h(x)$ at $x = 1$.
(4 pts.)

3.5 Let $f(x) = \ln \left(\frac{3x^2 - 2x - 4}{x^2 + x - 4} \right)^3$. Find $f'(2)$.
(4 pts.)

SCORE

Problem 4 20 points

4.1 Let $f(x, y) = y - e^{xy^2} + \sqrt{x^2 + 1}$.

(a) Find $f_x(1, 1)$.

(3 pts.)

(b) Find $f_y(1, 1)$.

(3 pts.)

4.2 Determine the value of a for which $f''(a) = 0$ if $f(x) = 2x^4 + 2x^3 - x$.

(5 pts.)

4.3 Suppose a production function is given by

$$P = \frac{2k\ell}{k^2 + \ell^2},$$

where P is product, ℓ is labor, and k is capital. Determine the marginal productivity with respect to ℓ when $k = 1$ and $\ell = 2$. (5 pts.)

4.4 Let $f(x) = xe^{x^2}$. Put the letter **T** in front of the item where the statement is true and the letter **F** in front of the item where the statement is false. Justification is not required.

(2 pts.)

(a) $f'(x) = (3x^2 + 1)e^{x^2}$

(b) $f''(x) = (4x^3 + 6x)e^{x^2}$

(c) $f'(1) = 4e$

(d) $f''(1) = 10e$

4.5 Let $f(x, y) = xy^2 + x^2y - \ln(x^2 - 2y)$. Put the letter **T** in front of the item where the statement is true and the letter **F** in front of the item where the statement is false. Justification is not required.

(2 pts.)

(a) $f(1, 0) = 0$

(b) $f_x(x, y) = y^2 + 2xy - \frac{2x}{x^2 - 2y}$

(c) $f_y(x, y) = 2xy + x^2 + \frac{2}{x^2 - 2y}$

(d) $f_x(1, 0) = 2$