

**Mahidol University International College****ICNS 103****Fundamental Mathematics****Midterm Exam****Saturday, 30 May 2015****10.00 - 11.50****40 points**

Directions Solve the following problems using the bottom of each page or any blank space for scratch-work. Answer the questions according to the instructions in each part. Write your name, ID number, section, and seat number in the space provided on each page. A calculator is NOT allowed for this exam.

SCORE**Problem 1 10 points**

1.1 Find the value (if any) of the following limits.

(a) $\lim_{x \rightarrow 3} \left(\frac{x^3 - 9x}{x^3 - 5x^2 + 6x} \right)$ (1 pt.)

(b) $\lim_{x \rightarrow 5} \left(\frac{x^2 - 25}{x^2 - ax + 15} \right)$ where $a = \lim_{x \rightarrow \infty} \frac{(2x)^3 - 4x^2 + 5}{(x+4)(x-5)^2}$ (2 pts.)

1.2 Use the definition of continuity to determine whether the following function is continuous at 3. (2 pts.)

$$f(x) = \begin{cases} \ln(4 - x) + 2, & \text{if } x < 3 \\ \sqrt{x^2 - 5}, & \text{if } x \geq 3 \end{cases}$$

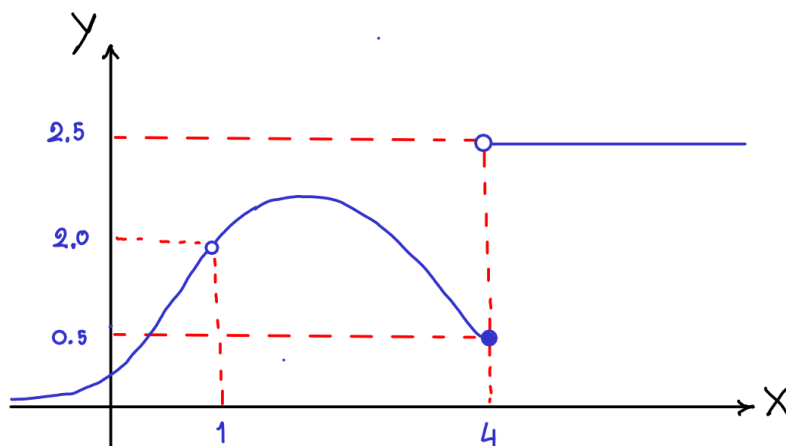
1.3 Find all points of discontinuity for $f(x) = \frac{x}{x((x + 1)^3 - 27)}$. (2 pts.)

1.4 From the graph of $y = f(x)$ on the next page, find the following:

(a) $\lim_{x \rightarrow 1} f(x) = \dots\dots\dots$ (1 pt.)

(b) $\lim_{x \rightarrow 4^-} f(x) = \dots\dots\dots$ (1 pt.)

(c) $\lim_{x \rightarrow \infty} 2f(x) = \dots\dots\dots$ (1 pt.)



Problem 2 10 points

SCORE

2.1 Fill in an appropriate response for each of the following: (2 pts.)

(a) $\frac{d}{dx}c = \dots\dots\dots$ (b) $\frac{d}{dx}x^n = \dots\dots\dots$

(c) $\frac{d}{dx}cf(x) = \dots\dots\dots$ (d) $\frac{d}{dx}(f(x) + g(x)) = \dots\dots\dots$

2.2 Using only the rules of differentiation provided in 3.1, find $f'(x)$ of the function: (4 pts.)

(a) $f(x) = \sqrt[3]{x^2} + \ln 2$

(b) $f(x) = x^e + e^2$

(c) $f(x) = \frac{x^2 - 3}{\sqrt{x}}$

(d) $f(x) = x^6(\sqrt{x} - \log 2)$

2.3 An equation of the tangent line to the curve of $y = x^3 + ax$ for some constant a at $x = 1$ is given by $y = 2x + b$ for some constant b . Find the value of a . (2 pts.)

2.4 State the definition of the derivative and use it to find $f'(x)$ where $f(x) = x - \frac{1}{x}$. (2 pts.)

SCORE

Problem 3 10 points

3.1 A manufacturer's average cost function in dollars per unit is given by $\bar{c} = 500 + 15q + 0.3q^2$.

(3 pts.)

(a) What is the total cost when 10 units are produced?

(b) What is the marginal cost when 10 units are produced?

(c) Interpret the answer in (b).

3.2 If $y = u^5 - 8u^2 + 2u - 1$ and $u = \sqrt{2x + 19}$, find $\frac{dy}{dx}$ when $x = -9$.

(3 pts.)

Multiple Choice

Circle the one alternative that best completes the statement or answers the question.

3.3 If $y = x^3 - 2x^2$, then the rate of change of y with respect to x when $x = -1$ is (1 pt.)

(A) $-\frac{7}{3}$

(B) -7

(C) $-\frac{1}{3}$

(D) 7

(E) -3

3.4 If $f(x) = (x + 1)^2(x + 2)^3$, then $f'(x) =$ (1 pt.)

(A) $(x + 1)(x + 2)^2(9x + 5)$

(B) $(x + 1)(x + 2)^2(3x + 2)$

(C) $6(x + 1)(x + 2)^2$

(D) $(x + 1)(x + 2)^2(5x + 7)$

(E) None of the above.

3.5 If $g(x) = \frac{x^2 - 5x + 2}{3x + 2}$, then $g'(x) =$ (1 pt.)

(A) $\frac{4 + 26x - 9x^2}{(3x + 2)^2}$

(B) $\frac{9x^2 - 26x - 4}{(3x + 2)^2}$

(C) $\frac{3x^2 + 4x - 16}{(3x + 2)^2}$

(D) $\frac{16 - 4x - 3x^2}{(3x + 2)^2}$

(E) None of the above.

3.6 From question 3.5 above, find $g'(1)$. (1 pt.)

(A) $\frac{21}{25}$

(B) $-\frac{21}{25}$

(C) $-\frac{9}{25}$

(D) $\frac{9}{25}$

(E) None of the above.

SCORE

Problem 4 10 points

4.1 Let $f(x) = e^{5x^2-9x-2} - \ln(4x - 7) + 3x^2$. Find $f'(2)$.

(4 pts.)

4.2 Let $f(x) = (2x + 1)e^{4x+3}$. Find $f''(-1)$.

(3 pts.)

4.3 Find an equation of the tangent line to the curve

$$x + xy + y^2 = 7$$

at the point $(1, 2)$.

(3 pts.)