Mahidol University International College
Trimester 1 Academic Year 2015-16

ICNS 103 Fundamental Mathematics Midterm Exam

Saturday, 24 October 2015 10.00 - 11.50 40 points

Directions Solve the following problems using the bottom of each page or any blank space for scratch-work. Answer the questions according to the instructions in each part. Write your name, ID number, section, and seat number in the space provided on each page. A calculator is NOT allowed for this exam. If not stated otherwise, all mathematical answers must be in simplest form.

SCORE

Problem 1 10 points

1.1 Find the value (if any) of the following limits.

(a) $\lim_{x\to-\infty}\left(\frac{-x^6+3x}{6x^3+x^6}-\frac{8}{x}\right)$ *(2 pts.)*

(b) $\lim_{x\to-5^-}\frac{25x^2-x^4}{x(5+x)}$ *(2 pts.)*

(c) $\lim_{x \to 4^-} \dfrac{|x-4|}{x-4}$ *(2 pts.)*

Recall: $|a| = \begin{cases} a, & \text{if } a \geq 0, \\ -a, & \text{if } a < 0. \end{cases}$

1.2 Find all possible values of k such that $\lim_{x \to k} g(x)$ exists where *(2 pts.)*

$$g(x) = \begin{cases} \frac{3}{x-2}, & \text{if } x < k, \\ x, & \text{if } x \geq k. \end{cases}$$

1.3 Find all points of discontinuity for $v(t)$ defined by *(2 pts.)*

$$v(t) = \begin{cases} 3t - t^2, & \text{if } t < 2, \\ t, & \text{if } t = 2, \\ \frac{t}{t-5}, & \text{if } t > 2. \end{cases}$$

Justify your answer.

SCORE

Problem 2 **10 points**

2.1 Let $f(x) = 3x^2 - 2x + 1$. Use the **definition** of the derivative to find $f'(x)$. No points will be awarded if the actual definition is not used. *(2 pts.)*

2.2 Complete the following basic differentiation rules: *(2 pts.)*

(a) $\frac{d}{dx}c =$ ..... (b) $\frac{d}{dx}x^n =$ .....

(c) $\frac{d}{dx}cf(x) =$ ..... (d) $\frac{d}{dx}\left(f(x) \pm g(x)\right) =$ .....

2.3 Find the derivative of $h(x) = \dfrac{2x^2 - 12x + 8}{\sqrt{x}}$ when $x = 4$, using only the basic rules of differentiation in (2.2). (No points will be awarded if other rules rather than those given in (2.2) have been used.) *(2 pts.)*

 scratch-work

2.4 Find an equation of the tangent line to the curve $y = \sqrt{16x} - \left(\frac{4}{x}\right)^5$ at $x = 4$. *(2 pts.)*

2.5 If the tangent line to the curve $y = ax^3 - 3x - 1$ is horizontal at $x = 3$, find all possible values of a. *(2 pts.)*

SCORE

Problem 3 **10 points**

3.1 A demand equation for a manufacturer's product is given by $p = 55 + 0.5q - 0.2q^2$ in dollars per unit. *(3 pts.)*

(a) What is the revenue function?

(b) What is the marginal revenue when 10 units of the output are sold?

(c) Interpret the answer in (b).

3.2 Find $f'(x)$ if $f(x) = (x-5)^{12}(x+6)^8$. Write your answer in the form $k(x-5)^m(x+6)^n(ax+b)$ for some constants k, m, n, a, and b. *(2 pts.)*

3.3 Find $g'(1)$ if $g(x) = \dfrac{x^3 - 4x^2 + 6}{7x - 5}$. *(2 pts.)*

3.4 If $y = u^3 - 6u^2 - 18u - 50$ and $u = \sqrt{3x - 12}$, find $\dfrac{dy}{dx}$ when $x = 6$. *(3 pts.)*

SCORE

Problem 4 **10 points**

4.1 Let $y = \ln w + w^2$ and $w = 5^{x \ln x}$. Find $\dfrac{dy}{dx}$ when $x = 1$. *(3 pts.)*

4.2 Find the slope of the tangent line to the curve $(x^2 + y)(3y + 5) + 2x = 35$ at the point $(1, 2)$. *(3 pts.)*

4.3 If $f(x) = 3xe^x - ax^2$ and $f''(0) = 3$, find the value of a. *(2 pts.)*

4.4 The average cost $\overline{c}$ (in thousands of dollars per unit) of q units of a product is given by

$$\overline{c} = \frac{1}{\ln(q^2 - 4q - 4) + 100}.$$

Find the total-cost function c and use the concept of marginal cost to approximate the cost in **dollars** of one additional unit of output when 5 units are produced. *(2 pts.)*