



**Mahidol University International College**  
**Trimester 2 Academic Year 2015-16**

<b>ICNS 103</b>	<b>Fundamental Mathematics</b>	<b>Midterm Exam</b>
<b>Saturday, 13 February 2016</b>	<b>08.00 – 09.50</b>	<b>40 points (35%)</b>

**Directions** Solve the following problems using the bottom of each page or any blank space for scratch-work. Answer the questions according to the instructions in each part. Write your name, ID number, section, and seat number in the space provided on each page. A calculator is NOT allowed for this exam. If not stated otherwise, all mathematical answers must be in simplest form.

<b>SCORE</b>

**Problem 1** 10 points

1.1 Find the value (if any) of the following limits.

(a)  $\lim_{x \rightarrow 1} \left( \frac{1}{x-1} - \frac{3}{x^2+x-2} \right)$  (2 pts.)

(b)  $\lim_{x \rightarrow 0^-} \frac{1}{x^2} \left( \frac{1}{2-x} - \frac{1}{2} \right)$  (2 pts.)

(c)  $\lim_{x \rightarrow -\infty} \left( \frac{2x}{(x-2)^2} - 3x^2 \right)$  (2 pts.)

1.2 Find all possible values of  $k$  such that  $\lim_{x \rightarrow 2} f(x)$  exists where (2 pts.)

$$f(x) = \begin{cases} \frac{3k}{x^2 + 4}, & \text{if } x < 2, \\ x - k, & \text{if } x \geq 2. \end{cases}$$

1.3 Determine whether the following function is continuous at 3. Justify your answer (2 pts.)

$$f(x) = \begin{cases} \frac{x-3}{x^2-9}, & \text{if } x < 3, \\ \frac{x}{x+15}, & \text{if } x \geq 3. \end{cases}$$

SCORE

**Problem 2** 10 points2.1 Let  $f(x) = (2x + 3)^2$ .

- (a) Use the **definition** of the derivative to find  $f'(x)$ . No points will be awarded if the actual definition is not used. (2 pts.)

- (b) Find  $f'(-1)$ . (1 pt.)

2.2 The limit

$$\lim_{h \rightarrow 0} \frac{(x+h)^{\frac{2}{3}} - x^{\frac{2}{3}}}{h}$$

represents the derivative of a function  $g$ . Determine a possible expression of  $g(x)$  and find the value of the limit at  $x = 8$ . (2 pts.)

2.3 Find  $\frac{dy}{dx} \Big|_{x=1}$  where  $y = 5x^3 + \frac{1}{\sqrt{x}}$ . (2 pts.)

2.4 Find all  $x$ -values on the curve  $y = (x + 1)(x^2 + 3x + 1)$  where the tangent line to the curve is horizontal. (3 pts.)

SCORE

**Problem 3** 10 points

3.1 The average cost  $\bar{c}$  for producing  $q$  units of a product is given by

$$\bar{c} = 0.01q^2 + 11 + \frac{1000}{q}.$$

What is the marginal cost when 10 units are produced? Interpret your result. (2 pts.)

3.2 Let  $g(x) = (\sqrt{4x} + 5x)((27x)^{2/3} - 5x^2)$ . Find  $g'(1)$ . Simplify your answer to an integer. (2 pts.)

3.3 A curve is given by  $y = 3 + \frac{14 - x}{x - 3}$ . Find an equation of the tangent line to the curve at the point where  $y = 13$ . (3 pts.)

3.4 Suppose that

$$y = \sqrt{u} + \frac{1}{\sqrt{u}} \quad \text{and} \quad u = (1 + 2x)^4 + 3(1 - 2x)^2.$$

Find  $\frac{dy}{dx} \Big|_{x=0}$ .

(3 pts.)

SCORE

**Problem 4** 10 points

4.1 Find the derivative of  $f(x) = e^{2x+1} + \log_2(3x^2 + 1)$ .

(2 pts.)

4.2 Given  $g(t) = 2^{3t-2}$ , find the value of  $g'(1)$ .

(2 pts.)

4.3 Find all values of  $x$  where the second derivative of  $y = \ln\left(\frac{x}{1-x}\right)$  is zero.

(2 pts.)

4.4 Find the rate of change of the first derivative of  $y = x^4 - 2 \ln x + \sqrt{x}$  at  $x = 1$ . (2 pts.)

4.5 Find the slope of the tangent line to the curve  $x^3 + 3x^2y - y^2 = 3$  at the point  $(1, 2)$ . (2 pts.)