Lecture 1 Exercises

May 1, 2023

1. Consider the function $f(x) = x^2 + c$ with $c \in \mathbb{R}$. For which values of c does the orbit of 0 converge? Prove your answer.

2. Let $f(x) = 4x(1-x)$ be the logistic map. Determine the lengths of the orbits for the following initial values x_0: $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$. Prove your answers.

3. Find all periodic points for each of the following maps and classify them as attracting, repelling, or neither.

   (a) $Q(x) = x - x^2$

   (b) $Q(x) = 2(x - x^2)$

   (c) $S(x) = \sin(x)$

4. Consider the linear maps $f_k(x) = kx$. Show that there are four open sets of parameters for which the behavior of orbits of f_k is similar. Describe what happens in exceptional cases.

5. For the function $f_\lambda = \lambda x(1-x)$ defined on $\mathbb{R}$, find all period 2 points.

6. Show that if x_0 lies on a cycle of period n, then

$$(f^n)'(x_0) = \prod_{i=0}^{n-1} f'(x_i).$$

   Conclude that

$$(f^n)'(x_0) = f(x^n)'(x_j)$$

   for $j = 1, ..., n-1$.

7. Consider the family $f_c(x) = x^2 + c$.

   (a) Compute explicitly the period 2 orbit.

   (b) Show that this orbit is attracting for $-5/4 < c < -3/4$.

8. Prove that $f_3(x) = 3x(1-x)$ on I is conjugate to $f(x) = x^2 - 3/4$ on a certain interval in $\mathbb{R}$. Determine this interval.