Lecture 2 Exercises

May 8, 2023

1. Show that $P(z) = z^2 - 2$ and $Q(z) = 2z^2 - 1$ are conjugate (that is, $P = \phi \circ Q \circ \phi^{-1}$, for some $\phi(z)$ of the form $az + b$).

2. Show that the map $P(z) = z^2 + c$ is conjugate to the map $Q(z) = \lambda z(1 - z)$ for any suitable λ and find the precise relationship between c and λ.

3. Show that $h(z) = z + 1/z$ is conjugate to the map $f \circ g$, where

$$f(z) = \frac{3z + 1}{z + 3}, g(z) = z^2.$$

4. Prove that if $|1 - \sqrt{1 - 4c}| < 1$, then the filled-in Julia set of $P_c(z) = z^2 + c$ has interior points.