Dynamics of Refinable Functions of Hyperbolic Type

MUIC Seminar May 2023

Wayne Lawton
Department of the Theory of Functions
Institute of Mathematics and Computer Science
Siberian Federal University, Krasnoyarsk, Russian Federation
wlawton50@gmail.com

Abstract

Let λ is an algebraic integer of degree $d \geq 2$, so $\lambda \notin \mathbb{Q}$ and

$$\mathbb{Z}[\lambda] := \mathbb{Z} + \lambda\mathbb{Z} + \cdots + \lambda^{d-1}\mathbb{Z}.$$

λ is a PV ; Salem number [PI] if the moduli of all other roots of its minimal polynomial are $< 1; \leq 1$. A generalized function φ on $\mathbb{R}$ is λ-refinable if $\int \varphi = 1$ and for some $c_k \in \mathbb{C}$, $\tau_k \in \mathbb{R}$

$$\varphi(x) = \sum_k c_k \, \varphi(\lambda \, x + \tau_k).$$

In 1939 Erdös [ER] proved that if λ is a PV number, $n = 2, \tau_1 = 0, \tau_2 = 1$ and $c_1 = c_2 = |\lambda|/2$ then φ is not integrable. In 1969 Kahane [KAH] extended his result for λ a Salem number, $\tau_k \in \mathbb{Z}$ and c_k finitely supported.
In 2015 [LAW2] we conjectured that Kahane's result holds for λ a PV number and $\tau_k \in \mathbb{Z}[\lambda]$ and motivated this conjecture in 2017 [LAW3]. In this talk we prove our conjecture, using the proven Lang's conjecture for tori [EG] , page 321, and formulate a multidimensional extension.

References 1

[CAS] J. W. S. Cassels, An Introduction to Diophantine Approximation, Cambridge Tracts in Mathematics and Mathematical Physics 45, Cambridge University Press, 1957.

[DFW] X. R. Dai, D. J. Feng, Y.Wang, Refinable functions with non-integer dilations, J. Funct. Anal. 250 (2007) 1–20.

[DAU] I. Daubecies, Orthonormal bases ofcompactly supported wavelets, Comm. Pure Appl. Math. 41 (1988) 909–996.

[ER] P. Erdös, *On a family of symmetric Bernoulli convolutions*, American J. Math. 61 (1939) 974–976.

References 2

[EG] J. H. Evertse and K. Gyory, Unit Equations in Diophantine Number Theory, Cambridge Univ. Press, 2015.

[FOUR] J. B. Joseph Fourier, J.B. Joseph, Théorie analytique de la chaleur, Paris: Firmin Didot, père et fils, Paris, 1822.

[HAAR] A. Haar Zur theorie der orthogonalen Funktionsysteme, Math Annal (1910)

[KAH] J. P. Kahane, Sur la distribution de certaines séries aléatoires. (French) Colloque de Th ´eorie des Nombres (Univ. Bordeaux, Bordeaux, 1969), Bull. Soc. Math. France, Mémoire. 25 (1971), 119–122.

References 3

[LAW1] Proof of the hyperplane zeros conjecture of Lagarias and Wang, The Journal of Fourier Analysis and Applications 14 (4) (2008) 588–605.

[LAW2] W. Lawton, Multiresolution analysis on quasilattices, Poincare Journal of Analysis & Applications 2 (2015) 37–52.

[LAW3] W. Lawton, Refinable functions with PV dilations, p. 177-188 in Approx. Theory XV: San Antonio 2016, (Gregory E. Fasshauer and Larry L. Schumaker, Ed.), Springer Proc. Math. & Stat. 201 (2017). arXiv:1605.06195v1

[OD] R. W. K. Odoni, Representations of algebraic integers by binary quadratic forms and norm forms from full modules of extension fields, Journal of Number Theory 10 (1978) 324–333.

References 4

[PI] C. Pisot, La répartition modulo 1 et nombres réels algébriques, Ann. Sc. Norm. Super. Pisa, II, Ser. 7 (1938) 205–248.

[SIEG] C. L. Siegel, Algebraic numbers whose conjugates lie in the unit circle. Duke Mathematical Journal 11 (1944) 597–602.

[VAN] B. L. van der Waerden, Algebra, Volume 2, Frederick Ungar, New York, 1970.